The Southland Economic Model

Technical Report

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Glossary

CES (Constant Elasticity of Substitution) An aggregating function used to combine two or more inputs into an aggregate quantity. Details can be found in Section 2.3.3 on page 8.

CET (Constant Elasticity of Transformation) A disaggregating function used to allocate a gross output between two or more possible outputs. Details can be found in Section 2.3.3 on page 8.

CGE (Computable General Equilibrium) A class of applied economic models often used to illustrate an economy's responses to changes in policy, technology or other external shocks. Typically, CGE models recognise a number of different types of economic agents (usually different types of industries, households, and government), conceptualised as either profit or utility maximisers. Optimisation algorithms are employed to determine the set of prices for all commodities and factors of production that would prevail subject to selected constraints (e.g. all commodity and factor markets clear and total income equals total expenditure for all agents).

CPI Consumer Price Index.

GDP (Gross Domestic Product) The total market value of goods and services produced in an economy after deducting the cost of goods and services utilised in the process of production, but before deducting allowances for the consumption of fixed capital.

Industry Value Added Value added summed according to aggregated industry groupings.

RoNZ (Rest of New Zealand) Once the study region or region of interest is chosen, all other regions from the multi-regional Social Accounting Matrix (SAM) are aggregated to form a single 'rest of NZ' region.

SAM Social Accounting Matrix.

System Dynamics A methodology for understanding certain kinds of dynamic systems. The methodology concentrates on mapping the feedback relationships between different components or relationships within a system, and simulating changes in systems over time.

1 Introduction

1.1 Background

The Southland Economic Model (often referred to in this report simply as 'the model') is a multisectoral and multi-regional dynamic economic model constructed as part of the Southland Economic Project. Although the principal purpose for creating the model is so that it can be used to test alternative freshwater management policies, the model is of a general structure and nature such that it could be used to test out a wide range of alternative futures and other types of policy scenarios.

The model is designed to imitate the core features of a Computable General Equilibrium (CGE) model. Among the advantages of these types of models are the whole-of-economy coverage, and the capture of not only indirect (i.e. the so-called upstream and downstream multiplier effects generated through supply chains) and induced (i.e. as generated through household consumption) economic consequences, but also the 'general equilibrium' (pricing) impacts.

While the model incorporates core features of a CGE model, it is important to note that it differs from a 'standard' CGE model in that it is a System Dynamics model formulated using finite differential equations. In our opinion, formulation of the model in this way more easily allows for the investigation of the *dynamic* implications of policies.

Once information is transformed into appropriate inputs and the model run, the model is able to produce a variety of indicators to help us evaluate the impacts of policies. Reporting indicators of multiple types (including GDP, industry value-added, employment and household consumption) and at multiple scales (Freshwater Management Unit, Territorial Local Authority, Southland Region, Rest of New Zealand) are generated. To help address uncertainty in the evaluation of policies, policy options can also be tested across a range of alternative assumptions about the future or 'reference economic futures'.

1.2 Objectives of this report

The purpose of this report is to provide a detailed technical documentation of the Southland Economic Model. By fully documenting all equations and data sources utilised in the model, we intend to make the model as transparent as possible.

It should nevertheless be noted that there are a variety of other reports which also describe work undertaken within the Southland Economic Project, much of this work is utilised directly within the Southland Economic Model. Reference can be made to the project website (waterandland.es.govt.nz/setting-limits/research/southland-economic-project) for further information.

2 Model Overview

2.1 Economic modelling context

2.1.1 Underlying theoretical basis

In economics, the general equilibrium theory of market behaviour and the extension of the theory into Computable General Equilibrium (CGE) with work by Johansen (1960) means that CGE models are now a well-established technique for describing economic behaviour. However, despite its widespread applicability and use, it is often criticised for an inability to properly deal with such things as time-path trajectories and out-of-equilibrium dynamics (Barker, 2004; Grassini, 2004; Scrieciu, 2007). The problem derives from the fact that the CGE model is concerned purely with the identification of steady states of economic equilibrium and has little or no functionality when tasked with establishing the time paths between steady states or the dynamics of non-equilibrium economic systems. In the real world, economies do not tend to have steady states of equilibrium but are constantly changing due to the influence of complex sets of destabilising forces. The Southland Economic Model incorporates elements of CGE modelling as an approach but uses them in a systems dynamics context which is a modelling framework used for analysing and simulating complex dynamic systems. One of the key aspects of turning a standard CGE model into a dynamic model is to explicitly model supply and demand relationships. The systems models can be viewed in terms of causal diagrams which incorporate feedback loops that tend to cause the system to naturally gravitate towards some equilibrium point. The establishment of these price-related balancing feedback loops is an essential component of this type of model.

Static CGE models have been converted to dynamic models in some applications by allowing key stocks, usually related to labour and capital resources, to be varied over time. The System Dynamics approach adopted here allows a similar extension. The model also incorporates a system of *information delays*, a concept from the System Dynamics approach, that serves to imitate the action of decisions made in the feedback process. In particular, the information delay seeks to incorporate gradual adjustments of beliefs that happen as a result of past experiences making similar decisions. This is done by incorporating a smoothing function that causes variables in the model to adjust gradually to current information, which means that recent information strongly influences their value, with the impact decreasing as time passes.

2.1.2 System Dynamics

Jay Forrester, at the Massachusetts Institute of Technology, developed System Dynamics during the mid-1950s (Forrester, 1961, 1969, 1971). System Dynamics is often described as a computer-aided modelling approach to policy analysis and design (e.g. Richardson, 2011). However, models constructed within System Dynamics programming languages are also frequently employed in problems that are not of a strict policy-orientation, for example design and engineering applications.

The System Dynamics approach relies specifically on using numerical methods (involving finite difference equations) to approximate solutions for ordinary differential equations along a path of successive 'time-steps'. Although these numerical approximation necessarily introduce some questions of accuracy, they are necessary in most cases, as the nonlinearity of the equations makes obtaining analytic solutions impossible. Furthermore, it significantly widens the scope of modelling exercises, enabling very complex systems to be represented within a computer simulation model, even by practitioners with no advanced mathematical training. Two popular graphical programming languages are now available for facilitating the construction of System Dynamics models, STELLA®and Vensim®. Both contain visual display and input and output features that enable users to easily grasp model structures, interactively run models and review results.

A core set of concepts employed in the development of this model is the distinction between *endogenous* variables: *stocks* or *auxiliaries*, and *exogenous* inputs: fixed model parameters (constants) or pre-determined time-varying inputs. In short, stocks are the independent variables within a simulation model that determine the condition of a system. These stocks accumulate (or dissipate) over time, and would continue to exist even if all relevant inflows and outflows (changes) to that stock ceased to exist. Stocks are endogenous as they are calculated within the model by solving the differential equations that describe the rates of change of the stocks. Note that the initial conditions for stocks are determined by the modeller, and are a special kind of exogenous input. By contrast, auxiliaries (sometimes also termed converters) can be thought of as 'intermediate steps' in the often complex functions defining rates of change of stocks. Auxiliaries are used to provide clarity to the modelling process by explicitly showing the steps required in the calculation of the rates of change or other output variables of interest. They also prevent the need for repetition in cases where the same calculations influence the rates of change of more than one stock. Auxiliaries are endogenous, as they are calculated explicitly at each time step from the values of the stocks and exogenous inputs in the model. The other model components are the fixed parameters (constants), and the time-varying inputs that are specified in advance (before the model is run). These are defined exogenous to the model, and can be varied by the modeller to investigate different scenarios.

2.2 The Southland Economic Model

2.2.1 Model structure

The basic structure of the Southland Economic Model is determined by the underlying regional Social Accounting Matrix (SAM) at its core (Smith *et al.*, 2015). The model considers two regions: the region of interest (Southland in this case) and the rest of New Zealand (RoNZ). For each region, the model describes the behaviour of representative agents (19 industry categories, 1 household, 1 enterprise, local government within each region, and central government). Each industry agent chooses the quantity and type of commodities (aggregated to 27 commodity categories) to produce, based on the prices of those commodities relative to the costs of production. household, enterprise, and government agents receive income from a variety of sources (e.g. wages and salaries, business profits, dividends, taxes, and transfers from other agents), and then allocate this income towards a variety of expenditure options (e.g. purchases of goods and services, savings, taxes, and transfers to other agents).

The model incorporates 'price' variables for all commodities and factors of production (i.e. types of labour and capital). These prices change in response to imbalances between supply and demand, and then 'nested' production functions allow the economy to react to these imbalances through substitution of demands and/or production between different types of commodities or factors. For example, if the demand for NZ-manufactured goods exceeds the supply, then the price of domestic goods will increase. This price increase (relative to foreign goods prices) will then lead to NZ-manufactured goods being substituted for goods produced overseas, thus reducing domestic demand and reducing prices. Similar substitution occurs in the factors and commodities used in production, and the region (within NZ) that the goods are demanded from. On the supply side, the relative prices determine how the supply of commodities and factors are split. For example, the supply of goods manufactured in NZ is split between the NZ and export markets depending on the relative prices in each market. So, if domestic goods prices increase, more of the goods produced will be allocated to the NZ market, which will increase domestic supply, thus decreasing prices.

The model incorporates the dynamics of economic growth by keeping track of stocks of capital held by each industry. Capital stocks accumulate via investments in new capital and are diminished via the ongoing process of depreciation.

The model also includes accounts that keep track of financial flows between NZ and the rest of the world (i.e. balance of payments). When the demand for NZ currency starts to outstrip supply this causes the exchange rate to rise. Changes in the exchange rate change the price of NZ goods relative to overseas goods, thus influencing demand and supply relationships. The model uses the NZ commodity prices along with exogenously specified world commodity prices to determine the supply and demand of exports and imports.

2.2.2 Model specifics

The model is divided into fourteen modules: households, governments, enterprises, industries, commodities, factors, labour, capital, savings & investment, municipal, primary and the rest of the world. Each module is described in detail in Section 3.

Much of the information for the model originates from a set of regional SAMs constructed for the financial year ending March 2007. The base year of the model, the regions available to be considered, and the minimum aggregation of industries and commodities are all determined by this process. A detailed report outlining the construction of a national and set of regional SAMs is also available (Smith *et al.*, 2015).

The SAMs were constructed for the 2006-07 financial year as at the time this was the latest year for which a comprehensive set of national accounts, including in particular the national Supply and Use Tables, was available from Statistics New Zealand.[1] Based on the timing of the multi-regional SAMs, we can interpret the time at which model simulations commence (i.e. $t = 0$) as being half way through that financial year, or 1 October 2006.

The Consumer Price Index (CPI) and Gross Domestic Product (GDP) indices are set to 1000 at $t = 0$. Following the convention in many CGE models, all prices are set equal to one for the reference year economic accounts. For our model this means that prices are all relative to the prices of the base year in 2007 NZ dollars ($\$_{2007}$). For example, if 20 kg of raw milk solids could be purchased for $\$_{2007}100$ during the base year, and the model shows the price increases from 1 at $t = 0$ to 1.5 at $t = 10$, then only 13.3kg of raw milk solids can be purchased for $\$_{2007}100$ at $t = 10$ or, put another way, that 20kg of raw milk solids would cost $\$_{2007}150$ at $t = 10$.

[1]Statistics New Zealand released a new set of Supply Use Tables, for the year ending March 2013. This data was utilised during model calibration.

2.3 Underlying assumptions

Some of the key assumptions that underpin the model's structure are as follows:

Agent Behaviours For each economic region, the economy can be described by the behaviour of a group of representative agents (industries, households, enterprises, local government, and central government). Industries are assumed to make choices about production and consumption solely based on the relative costs of inputs and values of production. Household, enterprise, and government agents receive income from a variety of sources (including from wages and salaries, business profits, dividends, taxes and transfers from other agents) and, in turn, allocate this income to a variety of expenditure options (purchases of goods and services, savings, taxes, and transfers to other agents).

Base Price Adjustment Time Lags As already explained, the model is a dynamic model able to describe not only the distribution of economic impacts across different sectors, but also the distribution of impacts through time. This extension is achieved essentially by creating price levels for all base commodities and factors of production (i.e. labour and capital). A key assumption is that all prices adjust upwards when supply is less than demand, and downwards when supply is more than demand. The parameters that determine the how far and how fast prices move in response to imbalances between supply and demand (α) are set via model calibration. The model does not at any stage attempt to compute the prices necessary to reach equilibrium (supply = demand) at any given time, instead the model calculates the changes in the base prices at each iteration (time step) and the new prices serve as inputs to the next iteration (time step). This creates time lags in base price adjustments in response to changes in supply or demand that depend on the α parameters. The way in which the time lags in base prices all interact over time contribute strongly to the dynamic behaviours captured by the model.

Other Adjustment Times in the Model There are many other variables of the model that do not adjust instantaneously. We can break these down into two subsets: variables that we believe should adjust almost instantaneously but that cannot be set as such within the model, and variables that we believe should adjust over a longer time. The price of 'composite' commodities or factors, i.e. commodities or factors that are made up of base commodities and factors, fits into the first category. If we were to calculate these prices instantaneously (at the same time step as the base prices and other variables in the model) this would create simultaneous equation loops. Solving these at each time step would be computationally very difficult, and furthermore it would be assuming that all agents have perfect information about the simultaneous actions of all other agents in the system. In this model we instead allow a delay of $\tau_{prices} = \Delta t$, so that prices update using the information from the previous iteration (time step) to determine current composite prices. Some variables in the model we believe should use information from the past over a longer range than one time step. For example, rather than using the instantaneous income to calculate expenditure, or even the income from the previous time step, this model smooths the income over a longer time, $\tau_{income} > \Delta t$. Other examples in the model include the industry production which is determined by considering the demand over the past $\tau_{industry} \approx 3$ months, and the interest rate and cash surplus, which are smoothed over the times $\tau_{interest}$ and $\tau_{casurplus}$, respectively.

Input Parameter Estimation The model incorporates a large number of other input parameters. Due to limitations in the availability of official statistics, and the significant resource required to develop alternative datasets, we have developed a full set of economic accounts for only a single year. These accounts, termed SAMs, are based predominantly on the 2006-07 financial year in accordance with the national supply and use tables released by Statistics New

Zealand. For the agricultural industries, the accounts were generated primarily from the financial information produced from case study farms modelled during 2015 (see Moran *et al.* 2017). Many of the input parameters are derived from the SAM (e.g. Constant Elasticity of Substitution (CES) and Constant Elasticity of Transformation (CET) share and scale parameters, proportion of income transferred overseas, commodity inputs required per unit of production), and are set as constant over a model run. Thus, it is assumed that relationships and behaviours exhibited during the 2006-07 financial year are a good approximation of relationships/behaviours in future modelled years.

Functional Forms Like many CGE models, the model repeatedly relies on the CES and CET functional forms to represent alternative demand (input) and supply (production) choices. 'Nested' CES and CET production functions allow the economy to react to imbalances between supply and demand in commodities/factors, through substitution of demand and/or production. These substitution possibilities occur in response to changes in relative prices. For example, a CES function describes the way in which demand for NZ-manufactured goods can be substituted for demand for goods produced overseas, if the price of domestic goods becomes too expensive relative to foreign goods. A separate CES function also describes the substitution between local-manufactured goods (i.e. produced within the same region) and the goods produced in the RoNZ. A CET function describes how the supply of goods produced in a region are split between the domestic and overseas markets, based on the relative prices, to maximise profit. While a separate CET function determines how the supply of goods to the domestic market is split between the local region and the RoNZ.

2.3.1 Conventions and notation used

Stocks are in bold font, with the first letter capitalised. Auxiliary (intermediate calculation) steps are named in all lower case italics. Variables with names in all capital letters and italics are exogenous inputs. Subscripts are used to indicate the dimensionality of a variable. For example, the stock $\mathbf{Pregdomcomm}_{sr,dr,c}$ denotes a price of regional domestic commodities specified by supply region (subscript 'sr'), demand region (subscript 'dr') and commodity type (subscript 'c'). Full information on subscripts is provided in Table 2.1.

As already indicated, two types of subscripts relate to regions, that is supply region (subscript 'sr') and demand region (subscript 'dr'). Occasionally within the model it is necessary to switch between these subscript types. The notation $sr \to dr$ in a subscript is used when the variables calculated for supply regions 1 and 2 respectively map to demand regions 1 and 2 (or vice versa when the subscripts are switched in the notation). The notation $DReg1 \leftrightarrow DReg2$ in a subscript is used when a quantity is transferred between regions i.e. the output from $DReg1$ goes to $DReg2$ and the output from $DReg2$ goes to $DReg1$.

2.3.2 Overview of computational method

The model is made up of a rate equation for each stock in the model that expresses how the value of that stock will change with time (known as a system of ordinary differential equations) in the form:

$$\frac{d}{dt}\mathbf{Stock} = rateofchange \tag{2.1}$$

The rates of change $rateofchange$ in this model can be (often nonlinear) functions of other stocks in the model at the current time or a past time (delays), as well as constant parameters and

Table 2.1 Subscripts used in the Southland Economic Model

Subscript indices	Description
$h = [CAP, LAB]$	Factors: Capital and Labour
$cap = [BuilC, NatC]$	Capital types: Built capital and Natural capital
$input = [FactsI, InterI]$	Input types: Factor inputs and Intermediate inputs
$g = [CentralG, LocalG]$	Governments: Central and Local
$sr = [SReg1, SReg2]$	Supply Regions: Region of interest and RoNZ
$dr = [DReg1, DReg2]$	Demand Regions: Region of interest and RoNZ
$i = [Ind1, Ind2, \ldots]$	Industries: see Table A.2 in Appendix A.1
$agi = [AgIn01, AgIn02, \ldots]$	Agricultural Industries: see Table A.4 in Appendix A.1
$IOag = [IOAg01, IOAg02, \ldots]$	IO Agricultural Industries: see Table A.5 in Appendix A.1
$ri = [ShBfDr, DaCaFm, \ldots]$	Report industry: see Table A.6 in Appendix A.1
$c = [Com1, Com2, \ldots]$	Commodities: see Table A.3 in Appendix A.1
$nct = [NatCap1, NatCap2, \ldots]$	Natural Capital types: see Table A.1 in Appendix A.1
$m = [Road, Rail]$	Transport margins: Road and Rail
$lt = [Capital, Land]$	Loan type: Capital and Land
$rt = [Normal, FinIntServ]$	Rate types: Normal and Financial Intermediate Services (see Table A.14 in Appendix A.1)
$ez = [Zone01, Zone02, \ldots]$	Economic zones: see Table A.12 in Appendix A.1
$ft = [FaTyp01, FaTyp02, \ldots]$	Farm types: see Table A.7 in Appendix A.1
$mt = [Miti01, Miti02, \ldots]$	Mitigation states: see Table A.8 in Appendix A.1
$mofa = [MoFa01, MoFa02, \ldots]$	Case study farms: see Table A.9 in Appendix A.1
$mf = [Hhld, Buss, \ldots]$	Municipal funding: see Table A.13 in Appendix A.1
$fmu = [Apar, Mata, \ldots]$	Freshwater Management Units: see Table A.10 in Appendix A.1
$ta = [South, Gore, \ldots]$	Territorial Authorities: see Table A.11 in Appendix A.1
$yrmt = [Year01, Year02, \ldots]$	Year Mitigated: see Table A.15 in Appendix A.1

time varying exogenous inputs. Due to the nonlinearity in the model, these equations cannot typically be solved explicitly to find $\mathbf{Stock}(t)$. However, these types of nonlinear dynamical systems arise almost ubiquitously in models of the real world and many methods have been developed to numerically approximate the solutions (values of $\mathbf{Stock}(t)$). Numerical methods for solving differential equations must be convergent, i.e. the numerical solution must converge to the exact solution (the error must go to zero) as the step size Δt goes to zero.

We have chosen to use Euler's method to numerically approximate the solution to the Southland Economic Model, as it is computationally easy to calculate, can deal with delays, and is available in most systems dynamics software. Euler's method transforms the rate equations for the stocks into finite difference equations numerically approximates the solution as:

$$\mathbf{Stock}(t + \Delta t) = \mathbf{Stock}(t) + (rateofchange) \times \Delta t \tag{2.2}$$

Euler's method uses a fixed time step and only one calculation (function value) is required per time step. Additionally, it is what is known as an *explicit* method which means that calculating the value of the stocks at a time step only requires knowledge of the values at the previous time step, as in Eq. 2.2. This simplicity does however mean that the numerical error does not decrease with step size as quickly as some other methods, so a smaller step size is required for numerical accuracy.

When we are evaluating different scenarios, there may be sudden changes in exogenous inputs to the model, often in the form of discontinuities. It is known that the Euler method can be numerically unstable in the case of sudden changes, leading to numerical solutions that oscillate when the actual solution does not. We must take care to examine the results near any discontinuities and reduce the step size further if there is any evidence of this unstable behaviour.

As discussed above, the step size choice is always a trade off between numerical accuracy, stability and computational time. After some investigation, we choose the time step $\Delta t = 0.0025$ years, which is approximately equal to a day. In the scenarios considered here this time step was found to produce stable numerical results for the Euler solution method we use to solve the model, whilst being able to be solved in a reasonable time. Additionally, in this model the time step is used explicitly to determine how fast various stocks adjust (e.g. the inflation rate, prices, industry production, and the GDP index). An adjustment time of a day is considered small enough to be realistic in these instances. In all cases, the adjustment times in the model ($\tau_{casurplus}$, τ_{income}, $\tau_{industry}$, $\tau_{interest}$, and τ_{prices}) must be greater than or equal to the time step, Δt, for the model to be numerically stable.

2.3.3 Constant elasticity of substitution and transformation functions

CES Like many CGE models, the model relies heavily on functions specified in a form that has become known as the CES functional form. The CES function is a particular type of aggregating function, which combines two or more types of goods, or two or more types of productive inputs into an aggregate quantity. As the name suggests, the function is characterised by the use of an elasticity of substitution, ϵ, which describes the percentage change in some quantity (e.g. the demand for a particular input) caused by a percentage change in the relative price of that quantity. The larger the elasticity, the greater the response to changes in price.

As an example, the CES production function for the demand for the ith composite good Q_i can be specified as:

$$Q_i = \gamma_i \left[\delta_i^m (M_i)^{\eta_i} + \delta_i^d (D_i)^{\eta_i} \right]^{\frac{1}{\eta_i}} \tag{2.3}$$

where M_i and D_i are the demand for the two input types (e.g. imported and domestic) available to produce good Q_i, and γ_i is the CES scale parameter. The CES input share coefficients for input types M_i and D_i are δ_m and δ_d, where $0 \leq \delta_i \leq 1$ and $\delta_i^m + \delta_i^d = 1$. Furthermore η_i is a parameter defined by the elasticity of substitution between inputs M_i and D_i:

$$\eta_i = \frac{\epsilon_i - 1}{\epsilon_i}, \qquad \eta_i \leq 1 \tag{2.4}$$

Now we can see that since $\eta_i \leq 1$, we have $1/(1 - \eta_i) \geq 0$, so as expected, when the price of an input p_i^d (or p_i^m) increases the demand for that input D_i (or M_i) decreases.

There are some special cases of the elasticity of substitution that are worth noting. Firstly, as $\epsilon \to \infty$, η goes to one and we have perfect (or linear) substitution. Conversely, in the extreme case where $\epsilon = 0$ (η goes to negative infinity) we have fixed shares of inputs, and Eq. 2.3 becomes a Leontief function. Finally, in the limit where η goes to 0, Eq. 2.3 becomes the well known Cobb-Douglas production function.

Let us assume that production choices are about maximising the value of outputs less input costs, and the demand for (composite) outputs produced are described by the CES production function above. The first-order conditions for this problem imply the following demand functions for the two input types:

$$\begin{aligned} M_i &= \left[(\gamma_i)^{\eta_i}\, \delta_i^m\, \frac{p_i^q}{p_i^m}\right]^{\frac{1}{1-\eta_i}} Q_i \\ D_i &= \left[(\gamma_i)^{\eta_i}\, \delta_i^d\, \frac{p_i^q}{p_i^d}\right]^{\frac{1}{1-\eta_i}} Q_i \end{aligned} \tag{2.5}$$

where p_i^m and p_i^d respectively denote the relative prices of input types M_i and D_i, and p_i^q is the price of the composite Q_i created by combining inputs M_i and D_i (before taxes or tariffs). The CES function-approach can be used either to (i) calculate the quantity of some 'composite' item produced from known quantities of inputs, or (ii) the relative demand for inputs of different types given the relative price of these inputs compared to the 'composite price' and given the total quantity of the composite item required.

CET The CET function is specified in the same form as the CES function (Eq. 2.3). However, this time instead of M and D representing two alternative types of inputs for the production of Q_i, D_i and E_i represent two alternative types of products into which the output Z_i can be transformed (e.g. export or domestic goods). The gross output must satisfy the equation:

$$Z_i = \theta_i \left[\xi_i^e (E_i)^{\phi_i} + \xi_i^d (D_i)^{\phi_i}\right]^{\frac{1}{\phi_i}} \tag{2.6}$$

Where D_i and E_i are the supply of the two possible outputs, and θ_i is the scale parameter for the CET function. The share parameters ξ_i^d and ξ_i^e have the properties $0 \leq \xi_i^d \leq 1$, $0 \leq \xi_i^e \leq 1$, and $\xi_i^d + \xi_i^e = 1$, as for the CES scale parameters.

As in the CES formulation, the elasticity of transformation, ψ, determines how sensitive the ratio of supply of outputs is to relative price changes, with larger ψ meaning that supply ratio responds more to price changes. However, the parameter derived from the elasticity for use in the CET function (Eq. 2.6) is:

$$\phi_i = \frac{\psi_i + 1}{\psi_i}, \qquad \phi_i \geq 1 \tag{2.7}$$

By assuming profit maximisation we get the supply of D_i and E_i to be:

$$\begin{aligned} E_i &= \left[(\theta_i)^{\phi_i}\, \xi_i^e\, \frac{p_i^z}{p_i^e}\right]^{\frac{1}{1-\phi_i}} Z_i \\ D_i &= \left[(\theta_i)^{\phi_i}\, \xi_i^d\, \frac{p_i^z}{p_i^d}\right]^{\frac{1}{1-\phi_i}} Z_i \end{aligned} \tag{2.8}$$

where p_i^z is the price of the gross output Z_i (including and taxes or tariffs), and p_i^d and p_i^e are the prices of transformed outputs D_i and E_i respectively. However, since $\phi_i \geq 1$, the quantity $1/(1-\phi_i) < 0$ and the relative quantity of D_i (or E_i) supplied will increases if the relative price of that product p_i^d (or p_i^e) increases compared to the gross output price p_i^z.

3 Modules

3.1 Household module

The household module can be conceptually separated into two parts, one dealing with household income and the other dealing with household expenditure. The full set of equations are available in Appendix B.1. A tree diagram that provides a summary of the different contributions to income and expenditure is shown in Figure 3.1.

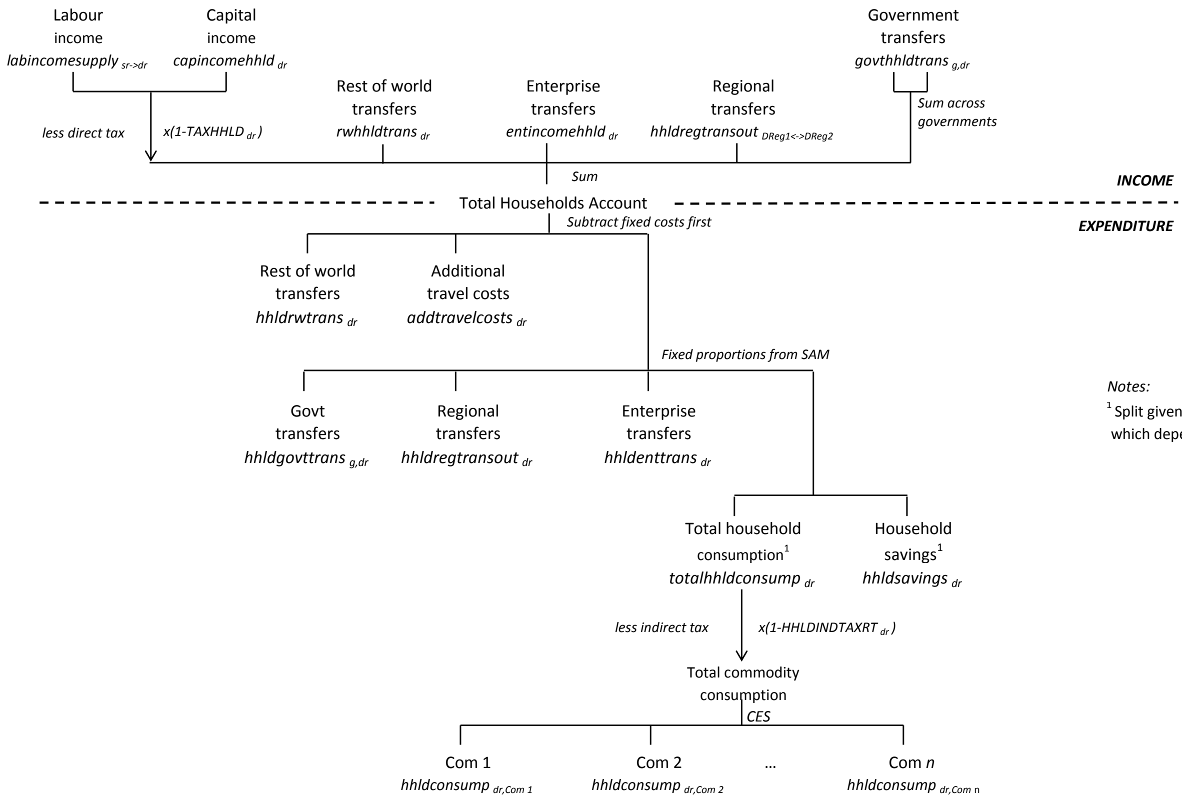

Figure 3.1 Tree diagram showing household income and expenditure. *CES*: Constant Elasticity of Substitution calculation. *SAM*: Social Accounting Matrix.

Starting with income, the principal sources of household income are factor payments to labour, $labincomesupply_{sr \rightarrow dr}$, capital income paid directly to households, $capincomehhld_{dr}$, and enterprise income that is transferred to households, $entincomehhld_{dr}$. Household incomes are further sourced from transfers from the rest of the world, $rwhhldtrans$, the government, $govthhldtrans_{g,dr}$, and interregional household transfers, $hhldregtransin_{dr}$. Altogether these sources of income, less direct taxes $hhlddirecttax_{dr}$ and income that is compulsory allocated towards funding new municipal wastewater schemes ($hhldloanpayments_{dr}$ and $vmunopex_{dr,c,mf=Hhld}$), constitute the available household income *actualhhldincome* (Eq. B.3):

$$actualhhldincome_{dr} = capincomehhld_{dr} + entincomehhld_{dr} + labincomesupply_{sr\rightarrow dr} + rwhhldtrans_{dr} + \sum_{g}(govthhldtrans_{g,dr}) + hhldregtransout_{DReg1\leftrightarrow DReg2} - hhlddirecttax_{dr} - hhldloanpayments_{dr} - \sum_{c} vmunopex_{dr,c,mf=Hhld}$$

Note that Southland Region's household funding of municipal wastewater treatment loans repayments, $hhldloanpayments_{dr}$ is simply the proportion of total loan payments attributed to households (Eq. B.25):

$$hhldloanpayments_{dr=DReg1} = \sum_{lt} totalloanpayments_{rt=Normal,mf=Hhld,lt}$$

$$hhldloanpayments_{dr=DReg2} = 0$$

Direct tax is charged on the capital income paid directly to households, $capincomehhld_{dr}$, and the factor payments to labour, $labincomesupply_{sr\rightarrow dr}$, at the rate $TAXHHLD_{dr}$ (Eq. B.23):

$$hhlddirecttax_{dr} = (capincomehhld_{dr} + labincomesupply_{sr\rightarrow dr}) \times TAXHHLD_{dr}$$

Capital and enterprise income are made up of local transactions, and transactions from outside the region (Eqs. B.4, B.5):

$$capincomehhld_{dr} = caplocalhhldtrans_{dr} + capreghhldtrans_{DReg1\leftrightarrow DReg2}$$

$$entincomehhld_{dr} = enthhldtrans_{dr} + entreghhldtrans_{DReg1\leftrightarrow DReg2}$$

To calculate the total value of labour income received by households within each region, *labincomeregion*, the model first determines the total value of income paid to labour within each region, *labincomedregion*, by multiplying the quantity of labour demanded in each region, $factorsu_{h=LAB,dr,i}$ by the current labour price, $preglabour_{dr}$. Note that an adjustment is made to account for a small proportion of labour supplied from outside of NZ thereby resulting in a transfer of labour income to the rest of the world. It is assumed that the proportion of labour income transferred to the rest of the world, $RWFACTRT$, remains constant with the base year (Eq. B.7):

$$labincomedregion_{dr} = \sum_{i}(factorsu_{h=LAB,dr,i}(1 - RWFACTRT_{h=LAB,dr})\,preglabour_{dr})$$

Labour income that is 'paid out' within a (demand) region can be allocated either to local households or households from outside of the region. In the model the relative shares are determined simply according to each (supply) region's contribution to total labour supply for a given (demand) region (Eq. B.6):

$$labincomesupply_{sr} = \sum_{dr}\left(\frac{labincomedregion_{dr}\,reglaboursupply_{sr,dr}}{\sum_{sr}(reglaboursupply_{sr,dr})}\right)$$

Incomes received by households from the rest of the world, $rwhhldtrans_{dr}$, are assumed to grow from the base year amount, $RWHHLDTRANSBS_{dr}$, at the same rate of growth as world Gross Domestic Product (GDP). The index of world GDP, $WORLDGDPINDX(t)$, is exogenous and thus can be adjusted for different simulations. In order to also allow for some adjustment in transfers in response to changes in the exchange rate, it is assumed that a share of rest of world transfers to households, i.e. $FCSHRWHHLDTRANS$, is calculated in foreign currency (Eq. B.9):

$$\begin{aligned} rwhhldtrans_{dr} &= RWHHLDTRANSBS_{dr}\, WORLDGDPINDX(t)\,(1 - FCSHRWHHLDTRANS_{dr}) \\ &\quad + RWHHLDTRANSBS_{dr}\, WORLDGDPINDX(t) \left(\frac{1}{\textbf{Exchangert}}\right) FCSHRWHHLDTRANS_{dr} \end{aligned}$$

Rather than using the auxiliary, *actualhhldincome*, to calculate expenditure in each time step based on instantaneous income in that same time step, this model uses the recognised household income **Rhhldincome**, which smooths the income over a longer time, τ_{income} (Eq. B.1):

$$\frac{d}{dt}\left(\textbf{Rhhldincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualhhldincome_{dr} - \textbf{Rhhldincome}_{dr}\right)$$

This has the advantage of avoiding stability issues due to simultaneous equations, as well as being more representative of household behaviour.

Turning now to household expenditure, six different 'sinks' for household income are recognised: household savings, *hhldsavings*, consumption of commodities, *totalhhldconsump*, transfers to the rest of the world, *hhldrwtrans*, transfers to government, *hhldgovttrans*, transfers to enterprises, *hhldenttrans*, and transfers to NZ households in other regions, *hhldregtransout*. This model also includes additional travel costs due to short term change such as a road outage, $addtravelcosts_{dr}$, which are described in Section 4.2.

In an analogous manner to transfers from the rest of the world to households, transfers from households to the rest of the world are assumed to vary from the base year in accordance with the change in world GDP and the exchange rate (Eq. B.8):

$$\begin{aligned} hhldrwtrans_{dr} &= HHLDRWTRANSBS_{dr}\, WORLDGDPINDX(t)\,(1 - FCSHHHLDRWTRANS_{dr}) \\ &\quad + HHLDRWTRANSBS_{dr}\, WORLDGDPINDX(t) \left(\frac{1}{\textbf{Exchangert}}\right) FCSHHHLDRWTRANS_{dr} \end{aligned}$$

Where $HHLDRWTRANSBS_{dr}$ is the transfers from households to the rest of the world in the base year, and $FCSHHHLDRWTRANS_{dr}$ is the proportion of transfers that are calculated in foreign currency.

The household transfers to the rest of the world, $hhldrwtrans_{dr}$, are subtracted from the recognised household income, **Rhhldincome**, first. In certain scenarios, e.g. road outages, there may be additional travel costs to the household, $addtravelcosts_{dr}$. These will be subtracted next. Once these expenditures have been subtracted, the exogenous constants $HHLDENTTRANSRT_{dr}$, $HHLDGOVTTRANSRT_{g,dr}$, and $HHLDREGTRANSRT_{dr}$ are used to define the shares of income reallocated from households to enterprises ($hhldenttrans_{dr}$), government ($hhldgovttrans_{g,dr}$), and other NZ households ($hhldregtransout_{dr}$), respectively. This gives Eqs. B.10, B.11, and B.12:

$$\begin{aligned} hhldenttrans_{dr} &= (\textbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \\ &\quad \times HHLDENTTRANSRT_{dr} \end{aligned}$$

$$hhldgovttrans_{g,dr} = (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \times HHLDGOVTTRANSRT_{g,dr}$$

$$hhldregtransout_{dr} = (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \times HHLDREGTRANSRT_{dr}$$

Of the remaining household income, $hhldtotal$, the proportion that is allocated to consumption, rather than savings, is determined by the household consumption rate, $hhldconsumprt$, such that, Eqs. B.13, B.14, and B.15:

$$hhldtotal_{dr} = \mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - hhldregtransout_{dr} - hhldenttrans_{dr} - \sum_{g}(hhldgovttrans_{g,dr}) - addtravelcosts_{dr}$$

$$totalhhldconsump_{dr} = hhldtotal_{dr}\, hhldconsumprt_{dr}$$

$$hhldsavings_{dr} = hhldtotal_{dr}\,(1 - hhldconsumprt_{dr}) + HHLDSAVINGADJUST_{dr}$$

Where an additional savings adjustment, $HHLDSAVINGADJUST_{dr}$, is added to account for the fact that in the base SAM household expenditure is greater than income. This is made possible through net increases in financial stocks.

It is generally recognised that household consumption is negatively correlated with changes in the real interest rate. When interest rates increase, people will be spending more on repaying mortgages and thus there will be less money available to spend on consumption of goods. In this model the parameter $CIRELASTICITY_{dr}$ controls the degree to which household consumption changes in response to changes in the real interest rate, which gives the relationship, Eq. B.16:

$$hhldconsumprt_{dr} = \left[\left(\frac{realinterestrt}{BASEREALINTERESTRT} - 1\right) CIRELASTICITY_{dr} + 1\right] \times BASECONSUMPRT_{dr}$$

Where $BASEREALINTERESTRT$ is the interest rate in the base year, and $BASECONSUMPRT_{dr}$ is the household commodity consumption rate in the base year.

Once the total value of income spent on consumption is determined, the households module also determines the proportion that is reallocated to government through the imposition of indirect taxes (e.g. GST). This is determined simply by multiplying the total value of consumption by a constant household indirect tax rate, $hhldindirecttaxrtadjusted_{dr}$. The default tax rate is calculated from base year data, but can be adjusted over time, if required under a scenario (Eqs. B.22, B.24).

$$hhldindirecttax_{dr} = totalhhldconsump_{dr} \times hhldindtaxrtadjusted_{dr}$$

$$hhldindtaxrtadjusted_{dr} = HHLDINDTAXRT_{dr} + HHLDTAXRTADJUST_{dr}(t)$$

The model then applies a constant Constant Elasticity of Substitution (CES) function procedure to determine household consumption of individual commodities. In short this process involves first determining the total demand for ‘composite’ commodities, $hhldcompcommd_{dr}$, by dividing the value of total consumption (minus the indirect taxes) by the applicable composite commodity price, $\mathbf{Phhldcc}_{dr}$ (Eq. B.21):

$$hhldcompcommd_{dr} = \frac{totalhhldconsump_{dr} - hhldindirecttax_{dr}}{\mathbf{Phhldcc}_{dr}}$$

Next, the first-order conditions for the CES problem enables a function to be generated that specifies the quantity of each commodity consumed, $hhldconsump_{dr,c}$, given the price of the particular commodity, $\mathbf{Pcompcommd}_{dr,c}$, relative to the composite price, $\mathbf{Phhldcc}_{dr}$, and the applicable CES scale parameters, γ_{dr}^{hhldc}, share parameters, $\delta_{dr,c}^{hhldc}$, and elasticity of substitution, η_{dr}^{hhldc} (Eq. B.19):

$$hhldconsump_{dr,c} = \left[(\gamma_{dr}^{hhldc})^{\eta_{dr}^{hhldc}} \, \delta_{dr,c}^{hhldc} \, \frac{\mathbf{Phhldcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}} \right]^{\frac{1}{1-\eta_{dr}^{hhldc}}} hhldcompcommd_{dr}$$

Finally using the same CES scale and share parameters, and elasticity of substitution, the quantity of composite commodities consumed can then be calculated (Eq. B.18):

$$qhhldcc_{dr} = \gamma_{dr}^{hhldc} \left[\sum_{c} \left(\delta_{dr,c}^{hhldc} \left(hhldconsump_{dr,c} \right)^{\eta_{dr}^{hhldc}} \right) \right]^{\frac{1}{\eta_{dr}^{hhldc}}}$$

Once we know the quantity of each composite commodity consumed, we can calculate the current household composite commodity consumption price using (Eq. B.17):

$$actualphhldcc_{dr} = \frac{\sum_c \left(\mathbf{Pcompcommd}_{dr,c} \, hhldconsump_{dr,c} \right)}{qhhldcc_{dr}}$$

The price stock $\mathbf{Phhldcc}_{dr}$ adjusts to the calculated current price $actualphhldcc_{dr}$ at the rate τ_{prices} (Eq. B.2):

$$\frac{d}{dt} \left(\mathbf{Phhldcc}_{dr} \right) = \frac{1}{\tau_{prices}} \left(actualphhldcc_{dr} - \mathbf{Phhldcc}_{dr} \right)$$

In order to have the prices respond almost instantaneously, the adjustment time is set to be equal to the time step $\tau_{prices} = \Delta t$. This means that the price adjusts within one time step and in the case where we use Euler's method to numerically solve the rate equations this is exactly equivalent to setting $\mathbf{Phhldcc}_{dr}(t) = actualphhldcc_{dr}(t - \Delta t)$.

3.2 Government module

The government module is very similar in structure to the households module (Section 3.1). The module can also be conceptually separated into equations dealing with income and equations dealing with expenditure. The full set of equations are available in Appendix B.2. A tree diagram that provides a summary of the different contributions to income and expenditure is shown in Figure 3.2.

On the income side, governments receive income mainly from direct and indirect taxes, $directtaxincome_{g,dr}$ and $indirecttaxincome_{g,dr}$. In addition to income from taxes, the governments receive some transfers of income from capital, $capgovttrans_{g,dr}$, enterprise, $entgovttrans_{g,dr}$, and households,

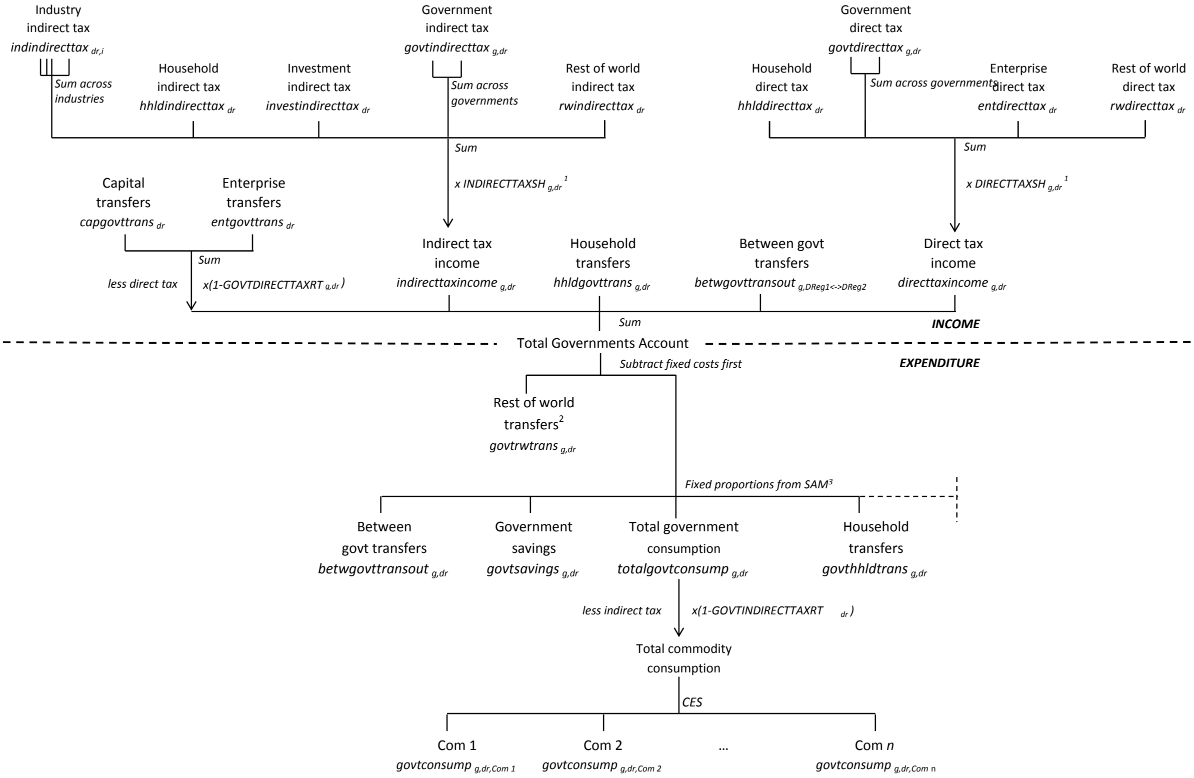

Figure 3.2 Tree diagram showing governments' income and expenditure. *CES*: Constant Elasticity of Substitution calculation. *SAM*: Social Accounting Matrix.
[1]How tax is split between local and central governments is exogenously determined.
[2]The amount that gets transferred to the rest of the world depends on the **Casurplus**.

$hhldgovttrans_{g,dr}$. The model also accounts for some minor financial transfers between government agents, $betwgovttransout_{CentralG \leftrightarrow LocalG,dr}$. These transfers are simultaneously a source of income for the receiving government agent, and a source of expenditure for the providing government agent. Also excluded from available government income, is the compulsory expenditure on new municipal wastewater schemes, both loan payments for new capital and operating expenditures (Eq. B.44). The actual government income is given by (Eq. B.28):

$$
\begin{aligned}
govtincome_{g,dr} =& directtaxincome_{g,dr} + indirecttaxincome_{g,dr} + capgovttrans_{g,dr} + entgovttrans_{g,dr} \\
& + hhldgovttrans_{g,dr} + \sum_{sr} betwgovttransin_{g,sr,dr} - govtdirecttax_{g,dr} \\
& - centralgovmunpays_{g,dr} + importtariffs_{g,dr}
\end{aligned}
$$

$$
\begin{aligned}
centralgovmunpays_{g,dr} =& \left(\sum_{lt} totalloanpayments_{rt=Normal,mf=Cgovt,lt} + \sum_{c} vmunopex_{dr,c,mf=Cgovt} \right) \\
& \times \frac{\mathbf{Rgovtincome}_{g,dr}}{\sum_{dr} \mathbf{Rgovtincome}_{g,dr}}
\end{aligned}
$$

Direct tax is charged on the capital and enterprise income transferred to governments at the rate

$GOVTDIRECTTAXRT_{g,dr}$ (Eq. B.31):

$$govtdirecttax_{g,dr} = \left[capgovttrans_{g,dr} + entgovttrans_{g,dr}\right] \times GOVTDIRECTTAXRT_{g,dr}$$

In the calculation of all direct and indirect tax transfers to governments, exogenous tax rates, $DIRECTTAXSH_{g,dr}$ and $INDIRECTTAXSH_{g,dr}$, are applied to determine how the tax income is split between local and central governments. Direct tax comes from taxes on the income of enterprises, $entdirecttax_{dr}$, households, $hhlddirecttax_{dr}$, the rest of the world, $rwdirecttax_{dr}$, and the governments itself, $govtdirecttax_{g,dr}$ (Eq. B.29):

$$directtaxincome_{g,dr} = \left[entdirecttax_{dr} + hhlddirecttax_{dr} + rwdirecttax_{dr} + \sum_{g}\left(govtdirecttax_{g,dr}\right)\right] \times DIRECTTAXSH_{g,dr}$$

While indirect taxes (including GST) come from the consumption of households, $hldindirecttax_{dr}$, investments, $investindirecttax_{dr}$, industries $indindirecttax_{dr,i}$, the rest of the world, $rwindirecttax_{dr}$, and the government, $govtindirecttax_{g,dr}$ (Eq. B.30):

$$indirecttaxincome_{g,dr} = \left[investindirecttax_{dr} + rwindirecttax_{dr} + hhldindirecttax_{dr} \sum_{i}(indindirecttax_{dr,i}) + \sum_{g}(govtindirecttax_{g,dr})\right] \times INDIRECTTAXSH_{g,dr}$$

The calculation of most of these indirect taxes is explained in other modules. In the case of industry indirect taxes, these are determined simply by multiplying the value of industry consumption by an indirect tax rate. The default tax rate, $INDINDIRECTTAXRT_{dr,i}$ is determined from the base social accounting matrix, but can be adjusted if necessary when applying different scenarios (Eqs. B.76, B.77):

$$indindirecttax_{dr,i} = \left[\sum_{c}\left(indconsump_{dr,i,c}\,\mathbf{Pcompdomcommd}_{dr,c}\right)\right] \times indindirecttaxrtadjusted_{dr,i}$$

$$indindirecttaxrtadjusted_{dr,i} = INDINDIRECTTAXRT_{dr,i} + INDTAXRTADJUST_{i}(t)$$

The final term in Eq. B.28 refers to income from import tariffs. Importantly, as import tariffs are already included as a category of indirect tax, this will normally be equal to zero, unless a scenario involves the application of special import tariffs over and above those normally charged.

Government expenditure includes transfers to households, $govthhldtrans_{g,dr}$, other government agents, $betwgovttransout_{g,dr}$, and the rest of the world, $govtrwtrans_{g,dr}$. The stock $\mathbf{Rgovtincome}_{g,dr}$, which is the recognised governments' (local and central) income, smoothed over the time τ_{income} (Eq. B.26):

$$\frac{d}{dt}\left(\mathbf{Rgovtincome}_{g,dr}\right) = \frac{1}{\tau_{income}}\left(govtincome_{g,dr} - \mathbf{Rgovtincome}_{g,dr}\right)$$

is used to determine the allocations of income to different expenditures.

The transfers from the government to the rest of the world, $govtrwtrans_{g,dr}$, vary from the base year amount $GOVTRWTRANSBS_{g,dr}$, depending on the current account surplus **Casurplus** according to the elasticity parameter $EGOVTTRANS$ (Eq. B.32):

$$govtrwtrans_{g,dr} = \text{sgn}\left(\mathbf{Casurplus}\right)\left|\mathbf{Casurplus}\right|^{EGOVTTRANS}\left(\frac{GOVTRWTRANSBS_{g,dr}}{\sum_g\sum_{dr}\left(GOVTRWTRANSBS_{g,dr}\right)}\right) + GOVTRWTRANSBS_{g,dr}$$

This expenditure is removed first, then the share of remaining income allocated to other areas is calculated (Eqs. B.34 - B.37):

$$govthhldtrans_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTHHLDTRANSRT_{g,dr}$$

$$betwgovttransout_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times BTWGOVTTRANSRT_{g,dr}$$

$$govtsavings_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTSAVRT_{g,dr}$$

$$totalgovtconsump_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTCONSUMPRT_{g,dr}$$

Once the total government consumption is calculated, the indirect tax paid on consumption is given by (Eq. B.43):

$$govtindirecttax_{g,dr} = totalgovtconsump_{g,dr}\, GOVTINDIRECTTAXRT_{g,dr}$$

As in the households module (Section 3.1) the CES function is used as a basis for allocating total government consumption expenditure among individual commodity types.

First the model determines the total demand for 'composite' commodities, $govtcompcommd_{g,dr}$, by dividing the total consumption (minus indirect taxes) by the applicable commodity price, $\mathbf{Pgovtcc}_{g,dr}$ (Eq. B.42):

$$govtcompcommd_{g,dr} = \frac{totalgovtconsump_{g,dr} - govtindirecttax_{g,dr}}{\mathbf{Pgovtcc}_{g,dr}}$$

Next, the consumption of individual commodities, $govtconsump_{g,dr,c}$ can be calculated from the demand for composite commodities, using a CES function that adjusts the particular commodity allocations based on the price of that commodity, $\mathbf{Pcompcommd}_{dr,c}$, relative to the composite price, $\mathbf{Pgovtcc}_{g,dr}$, using the CES scale parameters $\gamma^{govtc}_{g,dr}$, the CES share parameters $\delta^{govtc}_{g,dr,c}$, and the elasticity of substitution $\eta^{govtc}_{g,dr}$ (Eq. B.40):

$$govtconsump_{g,dr,c} = \left[(\gamma^{govtc}_{g,dr})^{\eta^{govtc}_{g,dr}}\, \delta^{govtc}_{g,dr,c}\frac{\mathbf{Pgovtcc}_{g,dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta^{govtc}_{g,dr}}} govtcompcommd_{g,dr}$$

The same scale and share parameters, and elasticity of substitution can then be used to calculate the quantity of composite commodities consumed (Eq. B.39):

$$qgovtcc_{g,dr} = \gamma^{govtc}_{g,dr}\left[\sum_c\left(\delta^{govtc}_{g,dr,c}\left(govtconsump_{g,dr,c}\right)^{\eta^{govtc}_{g,dr}}\right)\right]^{\frac{1}{\eta^{govtc}_{g,dr}}}$$

The actual price of government composite commodities is then calculated (Eq. B.38):

$$actualpgovtcc_{g,dr} = \frac{\sum_c \left(govtconsump_{g,dr,c}\,\mathbf{Pcompcommd}_{dr,c}\right)}{qgovtcc_{g,dr}}$$

And the price stock $\mathbf{Pgovtcc}_{g,dr}$ adjusts at rate τ_{prices} to meet this price (Eq. B.27):

$$\frac{d}{dt}\left(\mathbf{Pgovtcc}_{g,dr}\right) = \frac{1}{\tau_{prices}}\left(actualpgovtcc_{g,dr} - \mathbf{Pgovtcc}_{g,dr}\right)$$

3.3 Enterprise module

The enterprise module follows a similar form to the government module (Section 3.2) and the household module (Section 3.1). It can be broken up into income and expenditure sections, however, as enterprises do not consume commodities directly there is no consumption of commodity section. The full set of equations are available in Appendix B.3. A tree diagram that provides a summary of the different contributions to income and expenditure is shown in Figure 3.3.

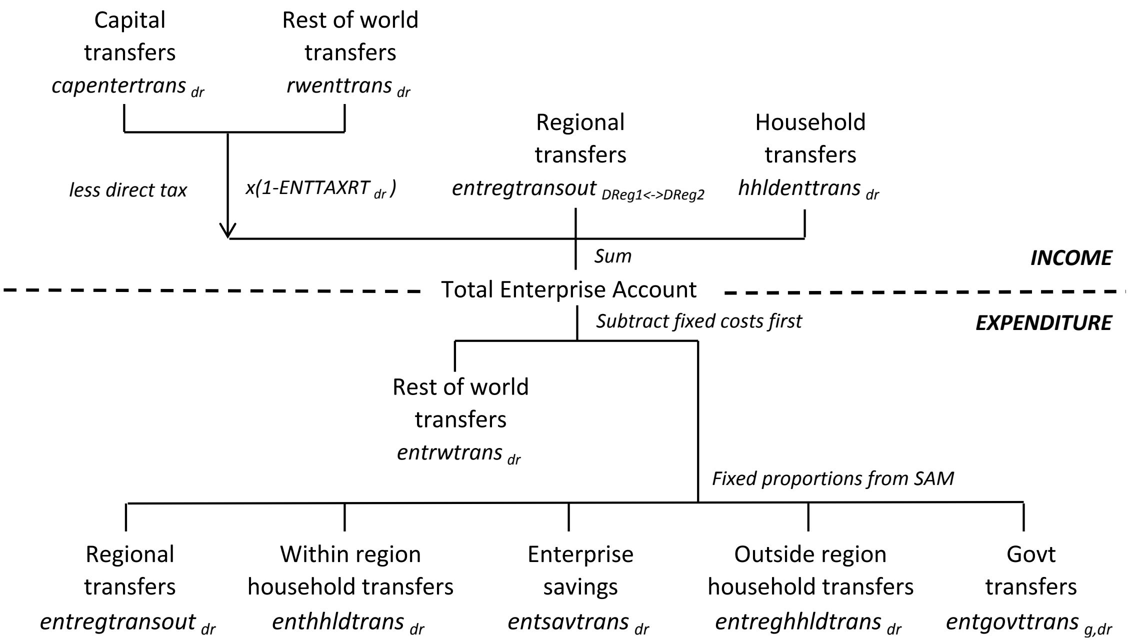

Figure 3.3 Tree diagram showing enterprise income and expenditure. *SAM*: Social Accounting Matrix.

The income of enterprises is made up of transfers of capital income, $capentertrans_{dr}$, transfers from households, $hhldenttrans_{dr}$, and transfers from the rest of the world, $rwenttrans_{dr}$. Additionally, this model includes enterprise income transfers between regions, which are simultaneously an expenditure for the providing region $entregtransout_{dr}$, and a source of income for the receiving region $entregtransout_{DReg1\leftrightarrow DReg2}$\$. This gives the equation for the total enterprise income (Eq. B.46):

$$\begin{aligned} actualenterincome_{dr} =& capentertrans_{dr} + hhldenttrans_{dr} + rwenttrans_{dr} \\ & + entregtransout_{DReg1\leftrightarrow DReg2} - entdirecttax_{dr} \end{aligned}$$

Where there is direct tax charged on capital and rest of world transfers only, at a rate $ENTTAXRT_{dr}$ (Eq. B.47):

$$entdirecttax_{dr} = (capentertrans_{dr} + rwenttrans_{dr}) \times ENTTAXRT_{dr}$$

Transfers to enterprises from the rest of the world, $rwenttrans_{dr}$, follow the same form as household transfers from the rest of the world (see Eq. B.9). Specifically, they are assumed to grow from the rate in the base year, $RWENTTRANSBS_{dr}$, following the GDP index $WORLDGDPINDX(t)$, with a fixed proportion, $FCSHRWENTTRANS_{dr}$, being set in foreign currency (Eq. B.48):

$$\begin{aligned} rwenttrans_{dr} = {} & RWENTTRANSBS_{dr}\, WORLDGDPINDX(t)(1 - FCSHRWENTTRANS_{dr}) \\ & + RWENTTRANSBS_{dr}\, WORLDGDPINDX(t) \left(\frac{1}{\mathbf{Exchangert}}\right) FCSHRWENTTRANS_{dr} \end{aligned}$$

The recognised enterprise income stock, $\mathbf{Renterincome}_{dr}$, follows the actual enterprise income, $actualenterincome_{dr}$, adjusting over the time τ_{income} (Eq. B.45):

$$\frac{d}{dt}\left(\mathbf{Renterincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualenterincome_{dr} - \mathbf{Renterincome}_{dr}\right)$$

Enterprise income is distributed to six different 'sinks' of expenditure: transfers to the rest of the world, $entrwtrans_{dr}$, enterprise transfers between regions, $entregtransout_{dr}$, transfers to governments, $entgovttrans_{g,dr}$, transfers to households within the region, $enthhldtrans_{dr}$, and outside the region, $entreghhldtrans_{dr}$, with the remaining transferred to savings, $entsavtrans_{dr}$.

As in the household and government modules, we subtract the transfers to the rest of the world first, with the amount calculated in a similar way (Eq. B.49):

$$\begin{aligned} entrwtrans_{dr} = {} & ERWTRANSBS_{dr}\, WORLDGDPINDX(t)(1 - FCSHENTRWTRANS) \\ & + ERWTRANSBS_{dr}\, WORLDGDPINDX(t) \left(\frac{1}{\mathbf{Exchangert}}\right) FCSHENTRWTRANS \end{aligned}$$

The remaining income is then allocated according to the proportions: $EREGTRANSRT_{dr}$, $EGOVTTRANSRT_{g,dr}$, $EHHLDTRANSRT_{dr}$, $ERHHLDTRANSRT_{dr}$, and $ESAVTRANSRT_{dr}$ (Eqs. B.50 - B.54):

$$entregtransout_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EREGTRANSRT_{dr}$$

$$entgovttrans_{g,dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EGOVTTRANSRT_{g,dr}$$

$$enthhldtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EHHLDTRANSRT_{dr}$$

$$entreghhldtrans_{dr} = [\mathbf{Renterincome}_{dr} - entrwtrans_{dr}] \times ERHHLDTRANSRT_{dr}$$

$$entsavtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times ESAVTRANSRT_{dr}$$

Where the proportions sum to exactly one, ensuring that all enterprise income is distributed to expenditure each time step.

3.4 Industries module

The industries module performs three primary functions: (1) Calculating the value of industry production within each region, (2) calculating the quantity of composite factors demanded by industries, and (3) determining the respective values of industry income and expenditure. The full set of equations are available in Appendix B.4.

Starting with the first, the model assumes that each industry has a desired level of production, $\mathbf{Desiredprod}_{dr,i}$, based on the value of industry sales. In turn the value of industry sales is calculated by first determining the value of commodity sales, including both domestic and export sales (Eq. B.59):

$$\begin{aligned} vcomdemand_{sr,c} = &\sum_{dr}(\mathbf{Pregdomcomm}_{sr,dr,c}\, regdomcommd_{sr,dr,c}) \\ &+ expcommodityd_{sr,c}\,\frac{\mathbf{Pexpcomm}_{sr,c}}{\mathbf{Exchangert}} \end{aligned}$$

Once the values of commodity sales are determined, these are allocated to industries based on each industry's relative share of total production, also termed supply coefficients, $supcoeff_{sr,i,c}$, thereby determining the total value of demand for industries' output, $vinddemandc_{sr,i,c}$ (Eq. B.58):

$$vinddemandc_{sr,i,c} = supcoeff_{sr,i,c}\, vcomdemand_{sr,c}$$

Rather than assuming that industries adjust production immediately to match the current value of sales, the model incorporates a time delay that acts to smooth industry production against short term fluctuations in commodity sales. This approach is well recognised in the System Dynamics literature and is termed 'psychological smoothing' (Forrester, 1961, p497). In short, psychological smoothing is a form of information delay that is recognised to occur whenever a decision that forms part of the feedback structure of a system is influenced by gradual adjustments of perceptions or beliefs. Thus, desired industry production is represented as a state of the system (i.e. stock) that either grows or declines based on the difference between it and the actual value of industry sales, as well as the assumed adjustment time for beliefs, $\tau_{industry}$ (Eq. B.55):

$$\frac{d}{dt}\left(\mathbf{Desiredprod}_{dr,i}\right) = \frac{1}{\tau_{industry}}\left(\sum_{c}(vinddemandc_{sr\rightarrow dr,i,c}) - \mathbf{Desiredprod}_{dr,i}\right)$$

The model further allows for the value of actual industry production, $actualprod_{dr,i}$, to vary from the desired level of production (see Eqs. B.60, B.61). The intention is to capture external influences on industries that prevent achievement of 'as normal' levels of production. Examples tested so far are disruptions in critical infrastructure services, such as water an electricity. This is achieved essentially by enforcing a maximum level of production when the operability of an industry, $OPERABILITY_{sr,i}(t)$, which is a scalar that varies between zero (complete disruption) and one (no disruption). This may not be very relevant to the types of scenarios for which the Southland Economic Model will be applied when evaluating water policy options, but could be relevant if the model is employed for other purposes such as examining the implications of a natural disaster.

Having determined the value of industry production, the quantity of industry production is determined simply by dividing by the unit cost of production, $unitcost_{dr,i}$. Given also the quantity of factors required per unit of production, $factinputshare_{dr,i}$, it is possible to determine the total demand for composite factors by industries, $compfactord_{dr,i}$ with (Eq. B.71):

$$compfactord_{dr,i} = \frac{maxprodsup_{dr,i}}{unitcost_{dr,i}} \frac{factinputshare_{dr,i}}{multifactorprod2_{dr,i}}$$

The denominator, $multifactorprod2_{dr,i}$, is the index of multi-factor productivity for each industry - see the factors and primary Modules for more detail. A scaling of $(1/multifactorprod2_{dr,i})$ is required in Eq. B.71, to account for the current level of productivity of the factor inputs. It is also important that the model uses $maxprodsup_{dr,i}$ (does not capture temporary adjustments to as-normal prudction) rather than $maxprod_{dr,i}$ in the calculation of composite factor demands. This means that industries 'hold on' to factor demands, at least in the short term, despite disruptions in operations. As rationale we would not expect, for example, firms to lay off staff immediately following a water supply disruption, even if those staff could not be effectively employed.

Nevertheless, it is necessary to also calculate the effective composite factor demands, $effectcompfactord_{dr,i}$, based on actual industry production, as this forms an input to the factors module (Section 3.6), and then the commodities module (Section 3.5), and the industry module (Section 3.4). This is calculated as (Eq. B.72):

$$effectcompfactord_{dr,i} = \frac{actualprod_{dr,i}}{unitcost_{dr,i}} \frac{factinputshare_{dr,i}}{multifactorprod2_{dr,i}}$$

The unit cost of production is made up of the unit cost of intermediate inputs and the unit cost of factors (Eq. B.73):

$$unitcost_{dr,i} = interinputunitcost_{dr,i} + factinputunitcost_{dr,i}$$

The contributions to unit cost are calculated based on the relative shares of intermediate inputs and factors necessary for production, $interinputshare_{dr,i}$ and $factinputshare_{dr,i}$, the relative prices of these inputs, $\mathbf{Pintinputs}_{dr,i}$ and $\mathbf{Pcfact}_{dr,i}$, and accounting for current state of multi-factor productivity and the imposition of indirect taxes on intermediate inputs (Eqs. B.74, B.75):

$$\begin{aligned} interinputunitcost_{dr,i} =& \left(interinputshare_{dr,i}\, \mathbf{Pintinputs}_{dr,i}\, (1 + indindirecttaxrtadjusted_{dr,i})\right) \\ & \times \left(\frac{1}{multifactorprod2_{dr,i}}\right) \end{aligned}$$

$$factinputunitcost_{dr,i} = \left(factinputshare_{dr,i}\, \mathbf{Pcfact}_{dr,i}\right) \times \left(\frac{1}{multifactorprod2_{dr,i}}\right)$$

The final components of the industries module are concerned with calculating industry income, $industryinc_{dr,i}$, and expenditure, $indexpendu_{dr,i}$. Income is calculated as the actual value of commodities supplied by each industry (Eq. B.68):

$$industryinc_{dr,i} = \sum_{c} (actualsupply_{sr \to dr,i,c})$$

where $actualsupply_{sr,i,c}$ is the minimum of the value of commodities supplied and the value of commodities demanded (Eq. B.69).

$$actualsupply_{sr,i,c} = \min[potentialsales_{sr,i,c}, vinddemandc_{sr,i,c}]$$

The value of commodities supplied, or in other words, potential sales supplied, $potentialsales_{sr,i,c}$, is determined simply by multiplying the quantity of commodities supplied, $indcommoditys_{sr,i,c}$, by the respective commodity prices, $\mathbf{Pcompcomms}_{sr,c}$ (Eq. B.70):

$$potentialsaless_{sr,i,c} = indcommoditys_{sr,i,c}\,\mathbf{Pcompcomms}_{sr,c}$$

The values for industry expenditure, on the other hand, are derived from the use of inputs to production, i.e. domestic commodities, imported commodities and factors, and the costs of those inputs, plus any indirect taxes paid (Eq. B.63):

$$\begin{aligned} indexpendu_{dr,i} = &\sum_{c}(domcommexpend_{dr,i,c} + importcommexpend_{dr,i,c}) \\ &+ \sum_{h}(factorsu_{h,dr,i}\,\mathbf{Pfact}_{h,dr,i}) + indindirecttax_{dr,i} \end{aligned}$$

When calculating expenditure on domestic commodities, $domcommexpend_{dr,i,c}$, and imported commodities,$importcommexpend_{dr,i,c}$, consideration is given to any economy-wide shortages in supply which may mean that industries are not able to purchase the full quantity of commodities demanded (Eqs. B.65, B.67, B.66, B.64, refer also to the commodities module for derivation of variables):

$$domcommexpend_{dr,i,c} = induseshare_{dr,i,c}\sum_{sr}(domcommodityuse_{sr,dr,c}\,pregdomcomminclmargin_{sr,dr,c})$$

$$domcommmodityuse_{dr,i,c} = \min[regcdomcomms_{sr,dr,c}, regcdomcommd_{sr,dr,c}]$$

$$induseshare_{dr,i,c} = \frac{indconsump_{dr,i,c}}{totalcomdemand_{dr,c}}$$

$$importcommexpend_{dr,i,c} = induseshare_{dr,i,c}\,importdemand_{dr,c}\,pimpcommnz_{dr,c}$$

Additionally, the model keeps track of any differences between industry income and expenditure, $\mathbf{Industrybalance}_{dr,i}$, as this surplus adds to industry 'profits' and thus needs to be considered in the calculation of Industry Value Added (Eqs. B.57, B.62):

$$\frac{d}{dt}\left(\mathbf{Industrybalance}_{dr,i}\right) = \frac{1}{\tau}\left(realindustrybalance_{dr,i} - \mathbf{Industrybalance}_{dr,i}\right)$$

$$\begin{aligned} realindustrybalance_{dr=DReg1,i} &= industryinc_{dr=DReg1,i} - indexpendu_{dr=DReg1,i} \\ &\quad - \textstyle\sum_{fmu}(indfencingcosts_{fmu,i} + indplancosts_{fmu,i}) \\ realindustrybalance_{dr=DReg2,i} &= industryinc_{dr=DReg2,i} - indexpendu_{dr=DReg2,i} \end{aligned}$$

Note that when calculating $realindustrybalance$ for region 1 (Southland), an adjustment is also made to account for additional industry expenditure on fencing, $indfencingcosts$ and farm plans, $indplancosts$.

3.5 Commodities module

The commodities module is the largest of all the modules in terms of the number of equations specified. However, many of the equations follow a repeated structure involving only different sets of quantities and prices. The full set of equations for the commodities module are available in Appendix B.5.

A visual representation of the key structure used to determine the regional domestic commodity supply and use (with associated domestic prices) is provided by the tree diagram in Figure 3.4.

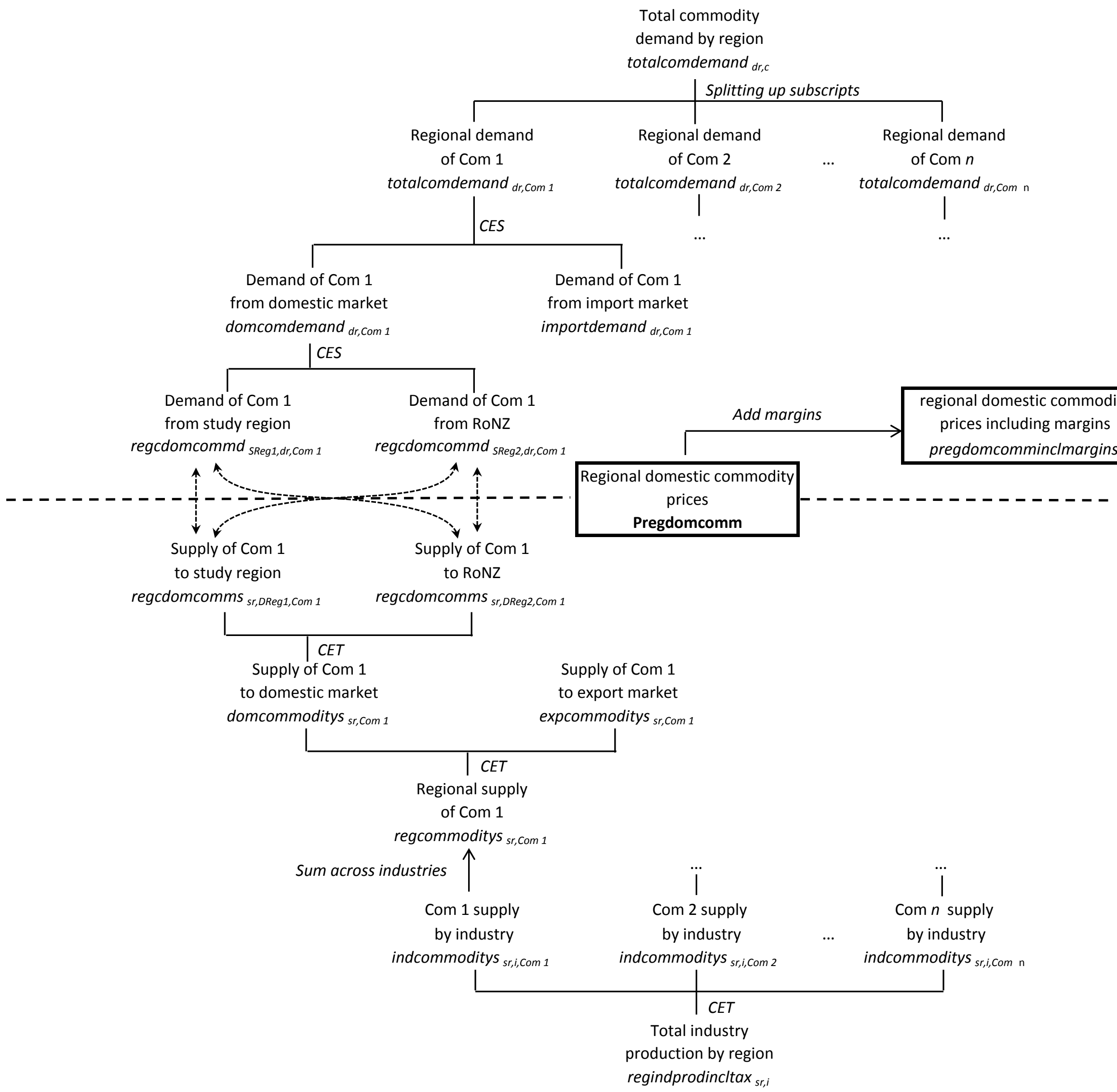

Figure 3.4 Tree diagram showing how the supply and demand of commodities are calculated. Here we just show the main structure for Commodity 1 (Com 1) due to space limitations, but the same Constant Elasticity of Transformation (CET) and CES transformations are applied to all commodities.

As can be seen, the commodities module sets in place a nested structure of CES and CET functions. The CES functions deal with the demand side of commodities, while the CET functions deal with the supply side. Towards the middle of the diagram the supply and demand sides meet, because the CES and CET structures are each used to specify the same set of quantity information, that is the the supply of each commodity c from each supply region sr to each demand region dr. These quantities are termed regional domestic commodity supply, $regcdomcomms_{sr,dr,c}$, from the supply side, and regional domestic commodity demand, $regdomcommd_{sr,dr,c}$, from the demand side.

The prices for these commodity flows, $\mathbf{Pregdomcomm}_{sr,dr,c}$ are the base prices of the commodities module, influencing the prices for all other quantities calculated elsewhere within the module. Like other base prices within the model, these are calculated by comparing the supply and demand of quantities, where the ratio of supply to demand is (Eq. B.100):

$$excessproduction_{sr,dr,c} = \frac{regcdomcomms_{sr,dr,c}}{regdomcommd_{sr,dr,c}}$$

The prices $\mathbf{Pregdomcomm}_{sr,dr,c}$ change following the rate equation:

$$\frac{d}{dt}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\right) = \left(\left(\frac{1}{excessproduction_{sr,dr,c}}\right)^{\alpha_c^{pregdomcomm}} - 1\right)\mathbf{Pregdomcomm}_{sr,dr,c}$$

From this equation we can see that the price is adjusted upwards if demand is greater than supply ($excessproduction_{sr,dr,c} < 1$), and downwards if supply is greater than demand ($excessproduction_{sr,dr,c} > 1$). The parameter $\alpha^{pregdommcomm}$ controls the magnitude of price adjustment rates in response to imbalances between supply and demand.

Once the base commodity prices have been calculated, the model provides an option to add an additional margin onto the prices, as may be necessary to reflect the circumstances of a scenario under investigation. For example, if $DMARGINSHOCKCOEF_{sr,dr,c}$ is the quantity of additional road transport margins charged per unit of commodity under a road outage scenario, and $pimproadmargins_{sr,dr}$ is the price per unit of road transportation services, the adjusted price, $pregdomcomminclmargin_{sr,dr,c}$, can be calculated as (Eq. B.449):

$$\begin{aligned} pregdomcomminclmargin_{sr,dr,c} =& DMARGINSHOCKCOEF_{sr,dr,c}\, pimproadmargins_{sr,dr} \\ &+ \mathbf{Pregdomcomm}_{sr,dr,c} \end{aligned}$$

See Section 4.2 for a description of how these margins are calculated.

Now, turning back to the supply of and demand for domestic commodities, i.e. $regcdomcomms_{sr,dr,c}$ and $regdomcommd_{sr,dr,c}$, we can trace along the branches of the CET and CES trees in Figure 3.4 to understand the way in which these quantities are calculated within the module.

3.5.1 Supply side

At the base of the supply tree in Figure 3.4, is the level of industry production within each region, $regindprodincltax_{sr,i}$. Production levels achieved by industries are controlled in the model by industries' effective use of composite factors, $effectivecompfactoru_{dr\to sr,i}$ as determined under the factors module (Section 3.6) (Eq. B.96):

$$regindprodincltax_{sr,i} = \frac{effectcompfactoru_{dr\to sr,i}}{factinputshare_{dr\to sr,i}}\, PRODSCALAR_{dr\to sr,i}\, multifactorprod2_{dr\to sr,i} \tag{3.1}$$

Here $factinputshare_{dr\to sr,i}$ specifies the ratio of composite factor inputs per unit of production, and $PRODSCALAR_{dr\to sr,i}$ and $multifactorprod2_{dr\to sr,i}$ are added to enable the estimated industry production levels to be adjusted to respectively account for indirect taxes and changes in multi-factor productivity. Note that as all of the variables on the right hand side of the equation are specified according to the demand region categories, these need to be first matched to the appropriate supply region for calculation.

A CET function, Eq. B.94, describes how the quantities of total industry production within each region, $regindprodincltax_{sr,i}$, are transformed into quantities of specific commodities produced by each industry within each region, $indcommoditys_{sr,i,c}$. This equation relies on scale and share parameters, $scalecommsup1_{sr,i}$ and $sharecommsup1_{sr,i,c}$ respectively. For non-primary industries, these parameters simply take on values defined exogenously. In the case of primary industries however, the scale and share parameters continuously update over time depending on changes in the structure of these industries. Thus, the scale and share parameters ($fiscalecomsup$ and $fisharecomsup$) are as calculated within the primary module (Eqs. B.94, B.142, B.143):

$$indcommoditys_{sr,i,c} = \left[(scalecommsup1_{sr,i})^{\phi^{comsup}_{sr,i}} sharecommsup1_{sr,i,c} \frac{\mathbf{Pcindustrys}_{sr,i}}{\mathbf{Pcompcomms}_{sr,c}}\right]^{\frac{1}{1-\phi^{comsup}_{sr,i}}} \times regindprodincltax_{sr,i}$$

$$scalecommsup1_{sr,i} = \begin{cases} fiscalecomsup_{IOag\to i} & \text{if} \quad i = \text{agricultural industry} \\ \theta^{comsup}_{sr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

$$sharecommsup1_{sr,i,c} = \begin{cases} fisharecomsup_{IOag\to i,c} & \text{if} \quad i = \text{agricultural industry} \\ \xi^{comsup}_{sr,i,c} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

The outcome of the CET function is that, depending on the strength of the assumed elasticity of transformation, for a given industry, the greater the price of a particular commodity relative to the composite price for all commodities supplied by that industry, the greater the proportion of industry output that will be devoted to supply of that commodity.

Having calculated $indcommoditys_{sr,i,c}$ we can also calculate the supply coefficients, $supcoeff_{sr,i,c}$, to be used in the industries module. These coefficients specify, of the total production of a particular commodity c in supply region sr, the proportion that is produced by industry i (Eq. B.95):

$$supcoeff_{sr,i,c} = \frac{indcommoditys_{sr,i,c}}{\sum_i (indcommoditys_{sr,i,c})}$$

Summing commodity production across all industries provides the total quantities of each commodity produced within each supply region, $regcommoditys_{sr,c}$ (Eq. B.93):

$$regcommoditys_{sr,c} = \sum_i (indcommoditys_{sr,i,c})$$

For each region, we are then able to split total commodity supply using a CET function into supply to the domestic market, $domcommoditys_{sr,c}$ (Eq. B.91), and supply to the export market, $expcommoditys_{sr,c}$ (Eq. B.90):

$$expcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsexp}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{pexpcommnz_{sr,c}}\right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c}$$

$$domcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsdom}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{\mathbf{Pcompdomcomms}_{sr,c}}\right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c}$$

Note that the share parameters in these CET functions sum to one: $\xi_{sr,c}^{commsdom} + \xi_{sr,c}^{commsexp} = 1$.

Finally, we use a CET function to split the domestic commodity supply between the two domestic regions (Eq. B.101):

$$regcdomcomms_{sr,dr,c} = \left[(\theta_{sr,c}^{commregs})^{\phi_{sr,c}^{regcom}} \xi_{sr,dr,c}^{commregs} \frac{\mathbf{Pcompdomcomms}_{sr,c}}{\mathbf{Pregdomcomm}_{sr,dr,c}} \right]^{\frac{1}{1-\phi_{sr,c}^{regcom}}} \times domcommoditys_{sr,c}$$

3.5.2 Demand side

We now turn to the demand side components of the commodities module, as depicted on the top half of Figure 3.4. Assuming for the time being that we already know the total demand in each region for each commodity, $totalcomdemand_{dr,c}$, these demands are first split into demands from the domestic market, $domcomdemand_{dr,c}$, and demands from the import market, $importdemand_{dr,c}$ via CES functions (Eqs. B.111, B.103):[2]

$$domcomdemand_{dr,c} = \left[(\gamma_{dr,c}^{commd})^{\eta_{dr,c}^{com}} \delta_{dr,c}^{commddom} \frac{\mathbf{Pperceivedcompcommd}_{dr,c}}{\mathbf{Pcompdomcommd}_{dr,c}} \right]^{\frac{1}{1-\eta_{dr,c}^{com}}} \times totaldemand_{dr,c}$$

$$importdemand_{dr,c} = \left[(\gamma_{dr,c}^{commd})^{\eta_{dr,c}^{com}} \delta_{dr,c}^{commdimp} \frac{\mathbf{Pperceivedcompcommd}_{dr,c}}{perceivedimportp_{dr,c}} \right]^{\frac{1}{1-\eta_{dr,c}^{com}}} \times totalcomdemand_{dr,c}$$

Note that the share parameters in these CES functions sum to one: $\delta_{dr,c}^{commddom} + \delta_{dr,c}^{commdimp} = 1$. Next, demands from the domestic market are split into demands from each individual region, $regdomcommd_{sr,dr,c}$, via another CES function (Eq. B.102):

$$regdomcommd_{sr,dr,c} = \left[(\gamma_{dr,c}^{commregd})^{\eta_{dr,c}^{regcom}} \delta_{sr,dr,c}^{commregd} \frac{\mathbf{Pcompdomcommd}_{dr,c}}{pregdomcomminclmargin_{sr,dr,c}} \right]^{\frac{1}{1-\eta_{dr,c}^{regcom}}} \times domcomdemand_{dr,c}$$

This completes all of the components of Figure 3.4, apart from the calculation of total commodity demands. Total commodity demands are a combination of the demands from households, investment, governments, and as intermediate inputs for industries. Some additional household consumption and margins are added for the case of infrastructure outages. This is described in Figure 3.5.

For each region, total demand for commodities is the sum of the demands from households, investment, governments and industries, with the first three determined under the respective

[2]Trade data often records the coexistence of imports and exports of the type of goods. To explain this anomaly, termed ‘cross-hauling’, it is proposed that such goods, despite fitting within the same classification, must in some way be different causing the goods to be *imperfectly* substitutable. Within economic modelling it tends to be substitution between imports and domestic goods, and between exports and domestic goods that is important. The degree of similarity between the these goods can be measured by a parameter such as the elasticity of substitution. The assumption about imperfect substitution between imports and domestic goods is called the Armington (1969) assumption, and it is often incorporated in Computable General Equilibrium (CGE) models via a CES function.

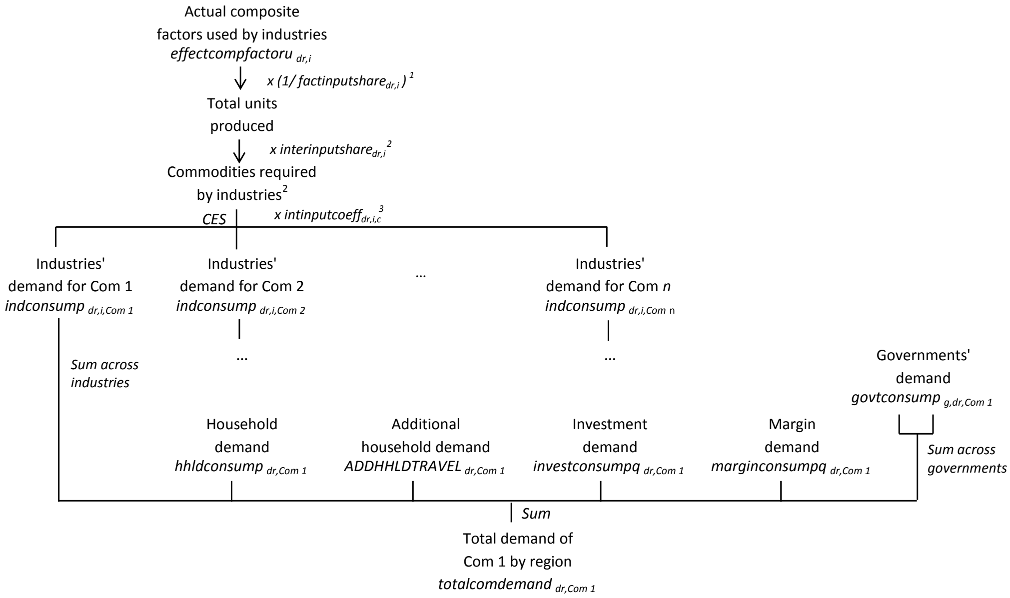

Figure 3.5 Tree diagram showing demand for commodities. For the final summation of demand we just show Commodity 1 (Com 1) due to space limitations, but the same transformations are applied to all commodities.

modules for each of these agents. Additionally, the model allows for net increases in demand for commodities that may be incurred due to the circumstances of a scenario (Eq. B.105):

$$\begin{aligned} totalcomdemand_{dr,c} = & \sum_{g}(govtconsump_{g,dr,c}) + hhldconsump_{dr,c} + investconsumpq_{dr,c} \\ & + \sum_{i}(indconsump_{dr,i,c}) + marginconsumpq_{dr,c} + totalfencingdemand_{dr,c} \\ & + totalplandemand_{dr,c} + loandemands_{dr,c} + OPEXDEMANDBYTIME_{dr}(t) \\ & \times WASTEMAP_{c} \end{aligned}$$

The term $totalfencingdemand_{dr,c}$ represents additional demands for fencing commodity as may be required under a scenario, while $totalplandemand_{dr,c}$ represents additional demand for commodities necessary for farm plan creation. The variable $marginconsumpq_{dr,c}$ captures additional consumption of road or rail transportation services for moving freight (all of domestic, import and export freight), while $ADDHHLDTRAVEL_{dr,c}$ is the net increase in consumption of transportation related commodities (e.g. petroleum, vehicle maintenance services) incurred by households. These latter two terms are unlikely to be relevant in water policy-related scenarios, but may be useful for other types of scenarios considered using the Southland Economic Model.

Commodities are used by industries as intermediate inputs of production. To calculate the demand from industries for commodities, $indconsump_{dr,i,c}$, required the following steps. First, we divide the total use of composite factors by industries, $effectcompfactoru_{dr,i}$, by the quantity of factors required per unit of production, $factinputshare_{dr,i}$. This gives us the total units produced by industries. We then multiply this total number of units by the intermediate inputs required per unit of production, $interinputshare_{dr,i}$, which gives us the overall demand for intermediate inputs from industries. Finally, the auxiliary variable $intinputcoeff_{dr,i,c}$, determines how this overall demand is split between the different commodity types. Putting this all together gives the equation (Eq. B.106):

$$indconsump_{dr,i,c} = intinputcoeff_{dr,i,c} \frac{interinputshare_{dr,i}}{factinputshare_{dr,i}} \times \begin{cases} compfactoru_{dr,i} & \text{for} \quad OPERABILITY_i = 1 \quad \text{or} \quad STOCKPILECOMM_c = 1 \\ effectcompfactoru_{dr,i} & \text{for} \quad OPERABILITY_i < 1 \quad \& \quad STOCKPILECOMM_c < 1 \end{cases}$$

The total consumption of each commodity across all industries is required, which we obtain simply by summing across all industries (Eq. B.107):

$$totalindconsump_{dr,c} = \sum_i (indconsump_{dr,i,c})$$

The coefficients in the above equation, $factinputshare_{dr,i}$, $interinputshare_{dr,i}$ and $intinputcoeff_{dr,i,c}$, are also derived from CES functions. At the top tier, $factinputshare_{dr,i}$ and $intinputcoeff_{dr,i,c}$ are derived by splitting the total units of inputs required per unit of production into two different categories, factors and intermediate inputs (Eqs. B.122, B.123):

$$factinputshare_{dr,i} = \left[(scalefi1_{dr,i})^{\eta^{fi}_{dr,i}} (sharefi1_{input=FactsI,dr,i}) \frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pcfact}_{dr,i}} \right]^{\frac{1}{1-\eta^{fi}_{dr,i}}}$$

$$interinputshare_{dr,i} = \left[(scalefi1_{dr,i})^{\eta^{fi}_{dr,i}} (sharefi1_{input=InterI,dr,i}) \frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pintinputs}_{dr,i}} \right]^{\frac{1}{1-\eta^{fi}_{dr,i}}}$$

Once again, we have the situation that the scale and share parameters for the CES function are defined exogenously for the non-primary industries, but determined within the primary module for the primary industries (Eqs. B.144, B.141):

$$scalefi1_{dr,i} = \begin{cases} fiscalefiinput_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{fi}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

$$sharefi1_{input,dr,i} = \begin{cases} fisharefiinput_{IOag \to i,input} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{fi}_{input,dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

We then calculate the share of intermediate inputs allocated to each commodity by the CES function, with scale and share parameters defined in a similar manner (Eqs. B.126, B.140, B.139):

$$intinputcoeff_{dr,i,c} = \left[(scalecominput1_{dr,i})^{\eta^{cominput}_{dr,i}} sharecominput1_{dr,i,c} \frac{\mathbf{Pintinputs}_{dr,i}}{\mathbf{Pcompcommd}_{dr,c}} \right]^{\frac{1}{1-\eta^{cominput}_{dr,i}}}$$

$$scalecominput1_{dr,i} = \begin{cases} fisalecominput_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{cominput}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

$$sharecominput1_{dr,i,c} = \begin{cases} fisharecominput_{IOag \to i,c} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{cominput}_{dr,i,c} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

3.5.3 Calculating prices

Export and Import prices In the model, the world commodity price of imports $PCOMMWORLDIMP_c(t)$ is exogenously determined in US dollars, and can be changed by the modeller to represent different future scenarios (forecasts). The commodity import price in NZ dollars, $pimpcommnz_{dr,c}$, is calculated simply as (Eq. B.108):

$$pimpcommnz_{dr,c} = \frac{PCOMMWORLDIMP_c(t)}{\textbf{Exchangert}} + pimportmargins_{dr,c} + pimporttariffs_c$$

where the margin component of the price $pimportmargins_{dr,c}$ is intended to capture short term additions of net margins charged on imports. For example, under a road outage scenario causing more costly transportation from ports, while $pimporttariffs_c$ covers additional tariffs on imported commodities that may be imposed under a scenario. Tariffs are calculated on an ad valorem (percentage) basis (Eqs. B.134, B.135):

$$pimporttariffs_c = \frac{PCOMMWORLDIMP_c(t)}{\textbf{Exchangert}}\, ADVALOREMIMPTARIFF_c(t)$$

$$importtariffs_{g=CentralG,dr} = \sum_c (pimporttariffs_c\, importdemand_{dr,c})$$

$$importtariffs_{g=LocalG,dr} = 0$$

For the purpose of the CES function which determines how total commodity demand is allocated among imported commodities and export commodities, a different imported goods price, $perceivedimportp_{dr,c}$, is used. It is calculated in a similar manner to $pimpcommnz_{dr,c}$, except that it also incorporates a further price adjustment that can be used to account for perceived negative qualities of imported commodities (for example additional time delays for delivery, poorer environmental standards in production). Although there is no actual money exchange associated with these 'prices', this can be used in scenarios where it is necessary for the model to show an increased preference for domestic goods over imported goods (or vice-versa with the application of a negative price adjustment) (Eq. B.138).

$$\begin{aligned} perceivedimportp_{dr,c} =& (1 + ADVALOREMIMPORTP_c(t))\, \frac{PCOMMWORLDIMP_c(t)}{\textbf{Exchangert}} \\ & + pimportmarginsdr, c + pimporttariffsc \end{aligned}$$

In terms of export prices, one option would be to assume that NZ producers are completely 'price takers' for all commodities. Prices received in NZ for exports would thus be equal simply to the world commodity price, $PCOMMWORLDEXP_c(t)$, divided by the exchange rate. However, to allow for some variation from this strict assumption, for example to account for lags in behaviour due to contracting or 'flooding' of markets, we have included a specific export commodity demand function in the model. Separate 'base' prices for export commodities, $\textbf{Pexpcomm}_{sr,c}$, are then calculated (Eq. B.84):

$$\frac{d}{dt}\left(\textbf{Pexpcomm}_{sr,c}\right) = \min\left(1, \left(\frac{1}{exportratio_{sr,c}}\right)^{\alpha^{pexpcomm}} - 1\right) \textbf{Pexpcomm}_{sr,c}$$

using the ratio of export commodity supply and demand, $exportratio_{sr,c}$ (Eq. B.89):

$$exportratio_{sr,c} = \frac{expcommoditys_{sr,c}}{expcommodityd_{sr,c}}$$

Under this approach, export demands grow from the amount in the base year at the rate of world GDP growth to the power of an adjustment parameter, $GDPPARAM$. Demands are also modified to account for changes in the relative price of NZ commodities as experienced by foreign purchases, $pexportcommd_{sr,c}$, compared to the world price. The exogenous parameter, $EXPORTP_c$, controls how responsive foreign demands are to changes in these relative prices as follows (Eq. B.97):

$$expcommodityd_{sr,c} = BASEEXPORTS_{dr \rightarrow sr,c} \left(\frac{PCOMMWORLDEXP_c(t)}{perceivedpexportdsr, c} \right)^{EXPORTP_c} \times WORLDGDPINDX(t)^{GDPPARAM_c}$$

Note that the price *perceivedpexportd* is used in Eq. B.97, rather than **Pexpcomm**. This is because even after accounting for differences in currencies, under certain scenarios the price experienced by foreign purchases for NZ commodities $pexportcommd_{sr,c}$ can vary from the price received by NZ producers for those same commodities, $pexpcommnz_{sr,c}$, due to the imposition of net additional transportation margins (Eq. B.98). The model also allows for a 'psuedo' price adjustment that can be used in scenarios to account for changing preferences by international consumers for New Zealand goods (Eqs. B.137, B.98, B.92):

$$perceivedpexportd_{sr,c} = (1 + ADVALOREMEXPORTP_c(t))\, pexportcommd_{sr,c}$$

$$pexportcommd_{sr,c} = \mathbf{Pexpcomm}_{sr,c} + pexportmargins_{sr,c}\mathbf{Exchangert}$$

$$pexpcommnz_{sr,c} = \mathbf{Pexpcomm}_{sr,c} \left(\frac{1}{\mathbf{Exchangert}} \right)$$

Composite prices All CES and CET-based functions that have been noted above for the commodities module depend on the input of a composite price. For example, the CES function which determines the apportionment of domestic commodity demand, $domcomdemand_{dr,c}$ into demand from specific regions, $regdomcommd_{sr,dr,c}$, requires as an input the composite demand price for all regions, $\mathbf{Pcommpdomcommd}_{sr,dr,c}$ (Eq. B.102). In a similar manner to the composite commodity consumption prices for households, governments and investment discussed in the respective modules above, all of the necessary composite prices in the commodities module are modelled as stocks that update to reflect changes in underlying quantities and prices.

In the case of the composite domestic commodity demand price, for example, the quantity of composite domestic commodity demand is calculated by combining the domestic commodity demand from source regions using a CES function (Eq. B.115):

$$qdomcommd_{dr,c} = \gamma^{commregd}_{dr,c} \left[\sum_{sr} \left(\delta^{commregd}_{sr,dr,c} (regdomcommd_{sr,dr,c})^{\eta^{regcom}_{dr,c}} \right) \right]^{\frac{1}{\eta^{regcom}_{dr,c}}}$$

Then, the total price for this quantity of composite demand can be calculated by multiplying the price for the base commodity demand (including margins), $pregdomcomminclmargin_{sr,dr,c}$, by the base commodity demand, $regdomcommd_{sr,dr,c}$, and then dividing this total value by the quantity (Eq. B.114):

$$actualpcdcd_{dr,c} = \frac{\sum_{sr}\left(pregdomcomminclmargin_{sr,dr,c}\, regdomcommd_{sr,dr,c}\right)}{qdomcommd_{dr,c}}$$

Once again, in order to have the prices respond almost instantaneously, the adjustment time is set to be equal to the time step, i.e. $\tau_{prices} = \Delta t$ (Eq. B.82):

$$\frac{d}{dt}\left(\mathbf{Pcompdomcommd}_{dr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpcdcd_{dr,c} - \mathbf{Pcompdomcommd}_{dr,c}\right)$$

The equations to calculate the remaining composite prices ($\mathbf{Pcindustrys}_{sr,i}$, $\mathbf{Pcompcommd}_{dr,c}$, $\mathbf{Pperceivedcompcommd}_{dr,c}$, $\mathbf{Pcompcomms}_{sr,c}$, $\mathbf{Pcompdomcomms}_{sr,c}$, $\mathbf{Pfiinputs}_{dr,i}$, $\mathbf{Pintinputs}_{dr,i}$) follow the same steps and are provided in Appendix B.5.

3.6 Factors module

Two types of factors are recognised in the Southland Economic Model, namely labour and capital. The factors module essentially deals with the supply of and demand for these factors, as well as the supply of and demand for composite factors. Figure 3.6 provides a visual representation of the factors module, while the full set of equations are available in Appendix B.6.

As shown at the top of the diagram, a key input to the factors module is the quantity of composite factors demanded by industries, $compfactord_{dr,i}$, as derived from the industries module (Section 3.4).

In order to split demands for composite factors, $compfactord_{dr,i}$, into demands for individual factors, i.e. labour and capital, the model once again relies on a CES function (Eq. B.155):

$$\begin{aligned} factorsd_{h,dr,i} = &\left[(factscalep1_{dr,i})^{\eta^{fact}_{dr,i}}\, factsharep1_{h,dr,i}\,\frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} compfactord_{dr,i} \\ &\times (1 - RWFACTRT_{h,dr}) \end{aligned}$$

As with other CES functions described above, the scale and share parameters are defined within the primary module for primary industries and exogenously for non-primary industries (Eqs. B.163, B.162):

$$factscalep1_{dr,i} = \begin{cases} fiscalefactorinput_{IOag\rightarrow i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{fact}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

$$factsharep1_{h,dr,i} = \begin{cases} fisharefactorinput_{IOag\rightarrow i,h} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{fact}_{h,dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

As part of the task of calculating factor demands, an adjustment is made to account for the proportion of factors supplied from abroad. The base SAM records a flow of income to the rest of the world associated with payments for labour. The relatively small proportion of the labour factor derived from overseas, $RWFACTRT_{h=LAB,dr}$, derived from these base year accounts is

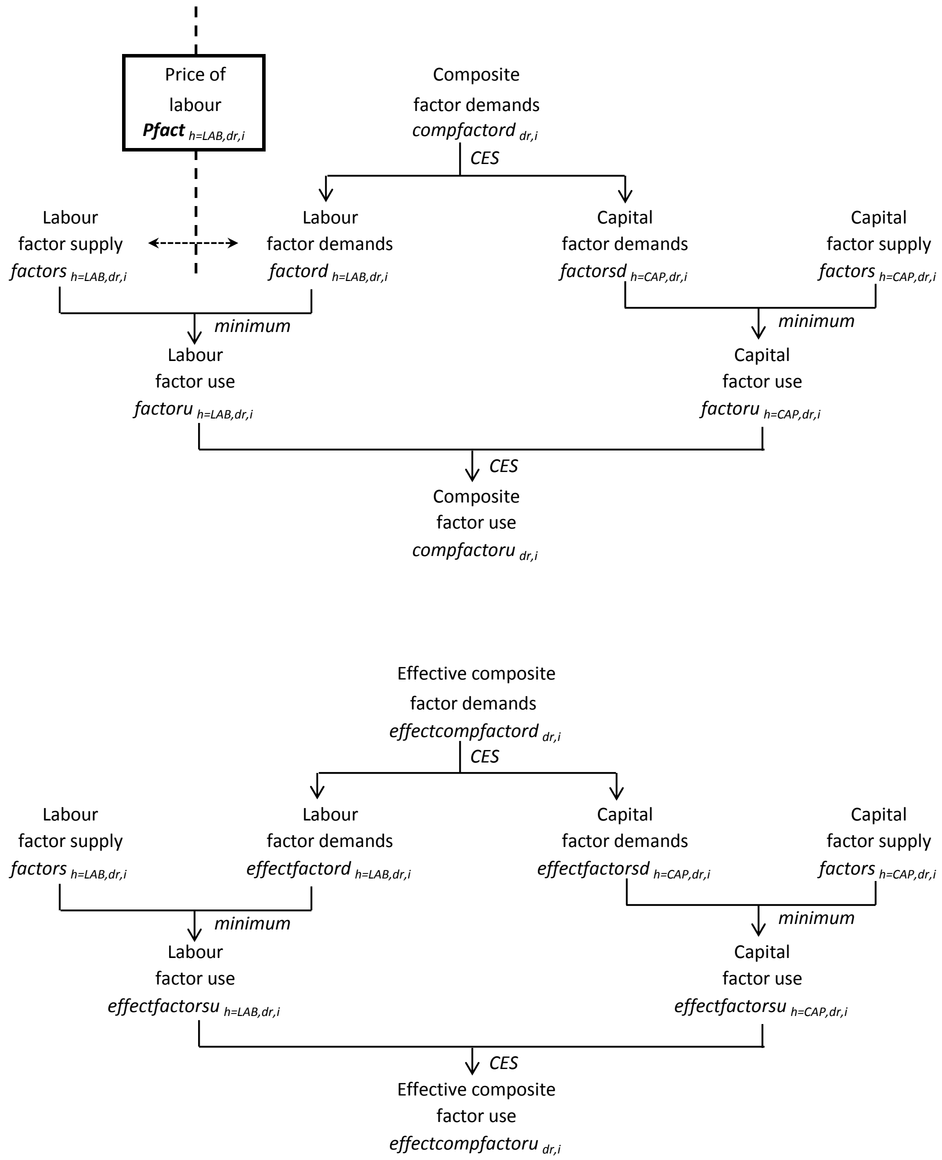

Figure 3.6 Tree diagram showing how the factors used and effective factors used are calculated.

assumed to remain constant for each model run. Note also that the proportion of capital factors that are supplied from overseas is equal to zero ($RWFACTRT_{h=CAP,dr} = 0$), also in accordance with base SAM.

The price appearing in the numerator of the above equation, $\mathbf{Pcfact}_{dr,i}$, is the price for composite factors and is calculated in the same manner as other composite prices within the model. That is, we take the sum of the value (i.e. price x quantity) of the individual items that make up the composite, and divide by the composite quantity achieved as determined by the CES function (Eqs. B.148, B.149):

$$\frac{d}{dt}\left(\mathbf{Pcfact}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcfact_{dr,i} - \mathbf{Pcfact}_{dr,i}\right)$$

$$actualpcfact_{dr,i} = \frac{\sum_h \left(factorsd_{h,dr,i}\,\mathbf{Pfact}_{h,dr,i}\right)}{qcompfactd_{dr,i}}$$

$$qcompfactd_{dr,i} = factscalep1_{dr,i} \left[\sum_h \left(factsharep1_{h,dr,i} \left(factorsd_{h,dr,i} \right)^{\eta^{fact}_{dr,i}} \right) \right]^{\frac{1}{\eta^{fact}_{dr,i}}}$$

Interestingly, the factor price contained in the denominator is a composite price in the case of capital, $\mathbf{Pfact}_{h=CAP,dr,i}$, but a base price in the case of labour, $\mathbf{Pfact}_{h=LAB,dr,i}$. In the model, capital is a composite item because it is further disaggregated into built capital and natural capital (see Section 3.8).

The composite capital demand price is computed as follows (Eqs. B.151, B.152):

$$\frac{d}{dt} \left(\mathbf{Pfact}_{h=CAP,dr,i} \right) = \frac{1}{\tau_{prices}} \left(actualpcapital_{dr,i} - \mathbf{Pfact}_{h=CAP,dr,i} \right)$$

$$actualpcapital_{dr,i} = \frac{capitaltyped_{cap=BuilC,dr,i}\, \mathbf{Pbuiltcap}_{dr,i}}{qcapitald_{dr,i}} + \frac{capitaltyped_{cap=NatC,dr,i}\, \mathbf{Pcompnaturalcapd}_{dr,i}}{qcapitald_{dr,i}}$$

$$qcapitald_{dr,i} = scalecc1_{dr,i} \left[\sum_{cap} \left(sharecc1_{cap,dr,i} (capitaltyped_{cap,dr,i})^{\eta^{cc}_{dr,i}} \right) \right]^{\frac{1}{\eta^{cc}_{dr,i}}}$$

where the built capital and composite natural capital demand prices, $\mathbf{Pbuiltcap}_{dr,i}$ and $\mathbf{Pcompnaturalcapd}_{dr,i}$ are provided from the capital module (Section 3.8). The supply of (composite) capital, $factorss_{h=CAP,dr,i}$, is also taken directly from the capital module (Eq. B.154):

$$factorss_{h=CAP,dr,i} = ccapitals_{dr,i}$$

Although in theory, labour could also be treated as a composite item, representing the combination of a variety of different labour skills, this has not been implemented in the present version of the model, with only one uniform labour type or skill recognised. For each industry, the model thus computes the total ratio of labour supply to labour demand, $labratio_{dr,i}$ (Eq. B.153):

$$labratio_{dr,i} = \frac{\sum_i (factorss_{h=LAB,dr,i})}{\sum_i (factorsd_{h=LAB,dr,i})}$$

Then, as with other base prices, the alpha parameter, α^{plab}, determines the degree to which the labour price changes in response to imbalances between supply and demand:

$$\frac{d}{dt} \left(\mathbf{Pfact}_{h=LAB,dr,i} \right) = \left(\left(\frac{1}{labratio_{dr,i}} \right)^{\alpha^{plab}} - 1 \right) \mathbf{Pfact}_{h=LAB,dr,i}$$

where the supply component for labour, $factorss_{h=LAB,dr,i}$ is taken directly from the labour module (Eq. B.154):

$$factorss_{h=LAB,dr,i} = adjustedindlaboursup_{dr,i}$$

The model further computes the maximum use of factors by considering the minimum of supply and demand (Eq. B.156):

$$factorsu_{h,dr,i} = \frac{\min[factorsd_{h,dr,i}, factorss_{h,dr,i}]}{1 - RWFACTRT_{h,dr}}$$

The use of the underlying labour and capital factors then allows us to calculate the composite factors used from the CES-composite function (Eq. B.157):

$$compfactoru_{dr,i} = factscalep1_{dr,i}\left[\sum_h \left(factsharep1_{h,dr,i}\left(\frac{factorsu_{h,dr,i}}{1-RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}}$$

The use quantities are important outputs of the factors module, being needed to calculate industry expenditure on factor inputs and, conversely, the income flows to capital and labour.

The remaining formulae within the factors module is concerned with calculating an auxiliary, termed effective composite factors used, $effectcompfactoru_{dr,i}$, which serves as an input to both the industry and commodities modules.

First, we use a CES function to split the effective composite demand into effective demand for labour and capital (Eq. B.159):

$$\begin{aligned}effectfactorsd_{h,dr,i} = &\left[(factscalep1_{dr,i})^{\eta^{fact}_{dr,i}}\ factsharep1_{h,dr,i}\ \frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} effectcompfactord_{dr,i}\\ &\times(1-RWFACTRT_{h,dr})\end{aligned}$$

Then the effective factors used are found from the minimum of the supply and the effective demand (Eq. B.158):

$$effectfactorsu_{h,dr,i} = \frac{\min[effectfactorsd_{h,dr,i}, effectfactorss_{h,dr,i}]}{1-RWFACTRT_{h,dr}}$$

where (Eq. B.160)

$$effectfactorss_{h,dr,i} = factorss_{h,dr,i} \times OPERABILITY_{sr\rightarrow dr,i}$$

And finally, the effective factors used are combined to give the effective composite factor use (Eq. B.161):

$$effectcompfactoru_{dr,i} = factscalep1_{dr,i}\left[\sum_h \left(factsharep1_{h,dr,i}\left(\frac{effectfactorsu_{h,dr,i}}{1-RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}}$$

The purpose of calculating this auxiliary is to ensure that when an industry is subject to a short term disruption in operability, the estimates of industry production and consumption of intermediate inputs, respectively calculated within the industry and commodities modules, can be appropriately scaled downwards, while allowing the use of capital and labour $compfactoru_{dr,i}$ to remain at the undisrupted level. The steps for calculating the effective demand and use of factors and composite factors follow the same steps as for the undisrupted case.

A final task of the factors module is to calculate the productivity indices for each industry, $multifactorprod2_{dr,i}$. For industries dealt with under the primary module, the stock **Agfarmsystemprod**, constantly updates to reflect the values of multi-factor productivity calculated at an industry-level under that module. For all other industries, the multifactor productivity indices take on the value defined by $mfpadjusted_{dr,i}$ (Eqs. B.165, B.145, B.164, B.166)

$$multifactorprod2_{dr,i} = \begin{cases}\mathbf{Agfarmsystemprod}_{dr=DReg1,i} & \text{if} \quad i = \text{agricultural industry}\\ mfpadjusted_{dr,i} & \text{if} \quad i = \text{non-agricultural industry}\end{cases}$$

$$\frac{d}{dt}\left(\mathbf{Agfarmsystemprod}_{dr=DReg1,i}\right) = \frac{actualagriprod_{dr=DReg1,i} - \mathbf{Agfarmsystemprod}_{dr=DReg1,i}}{TIME\ STEP} \times (1 - OVERRIDEMAP_i)$$

$$actualagriprod_{dr,i} = \begin{cases} agriprodrt_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ 0 & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

$$mfpadjusted_{dr,i} = MULTIFACTORPROD_{dr,i}\ \left(1 + ADJUSTRATE)^{Time} - 1\right)$$

3.7 Labour module

The labour module tracks the amount of labour available, and apportions it to different regions and industries. The full set of equations are available in Appendix B.7.

$\mathbf{Indlaboursup}_{dr,i}$ defines the stock of labour available within each region to each individual industry. Recognising that different industries require different skill sets and therefore, shortages in the supply of labour will not necessarily be simply filled from surpluses in other industries, only a portion of labour held within each industry's stock, $labtoreallocate_{dr,i}$, is available annually for reallocation. In turn, the total quantity of labour available for reallocation each year is apportioned to industries on a pro-rata basis according to each industry's relative demands. New labour that enters the system, $netincreaselabdr$, is also apportioned to industries (Eqs. B.167, B.169, B.168, B.170, B.171)

$$\frac{d}{dt}\left(\mathbf{Indlaboursup}_{dr,i}\right) = newlabsupply_{dr,i} + reallocatedlab_{dr,i} - labtoreallocate_{dr,i}$$

$$labtoreallocate_{dr,i} = \mathbf{Indlaboursup}_{dr,i}\, MAXREALLOCATERT$$

$$reallocatedlab_{dr,i} = \sum_i labtoreallocate_{dr,i} \times \frac{factorsd_{h=LAB,dr,i} \times \frac{1}{LABINDEX_{dr,i}(t)}}{\sum_i \left(factorsd_{h=LAB,dr,i} \times \frac{1}{LABINDEX_{dr,i}(t)}\right)}$$

$$newlabsupply_{dr,i} = netincreaselab_{dr} \frac{\mathbf{Indlaboursup}_{dr,i}}{\sum_i \mathbf{Indlaboursup}_{dr,i}}$$

$$netincreaselab_{dr} = \frac{1}{\tau}\left(\sum_{sr} reglaboursupply_{sr,dr} - \sum_i \mathbf{Indlaboursup}_{dr,i}\right)$$

Note that before the determined supply of labour available to each industry is used within the rest of the module, there is an allowance for an additional productivity adjustment. Although this adjustment is not used in the base case, it can be useful for implementing scenarios where productivity gains are specific to just one factor type. The final 'effective' quantity of labour is termed $adjustedindlaboursup_{dr,i}$ (Eq. B.177):

$$adjustedindlaboursup_{dr,i} = \mathbf{Indlaboursup}_{dr,i}\, LABINDEX_{dr,i}(t)$$

The total annual quantity of labour supply available to each region, from each region, $reglaboursupply_{sr,dr}$, is defined in base-year dollar terms and is equal to the value of labour income that could be received over a year. This is generated by converting the quantity of available workers for job positions (which will generally be greater than the number of persons available for work as a single person can have two jobs), by the conversion factor $LSFCONVERT$ (Eq. B.173):

$$reglaboursupply_{sr,dr} = reglabourest_{sr,dr}\, LSFCONVERT_{dr}$$

In turn, the number of available workers for job positions in each region is generated by taking the labour force (number of working aged persons multiplied by the participation rate) and multiplying by the average number of positions held by each member of the labour force, $MECRATIO_{sr}$. An adjustment is however first required for the proportion of the labour force not able to be considered available due to reasons such as imprisonment, $LABFORCEADJUST_{sr}$, and further for the proportion for which it is assumed employment can never be achieved, termed $FUNEMPLOYRT$. An adjustment is also required for the net quantity of available workers for job positions that do not work in the same region as their location of residence, $MECTRANSOUT_{sr}$ (Eqs. B.175, B.176, B.174, B.173):

$$labourforce_{sr} = (1 - LABFORCEADJUST_{sr})\, WORKINGAGEPOP_{sr}(t)\, PARTICIPATIONRT_{sr}(t)$$

$$unavailablelab_{sr} = (1 - LABFORCEADJUST_{sr})\, WORKINGAGEPOP_{sr}(t)\, FUNEMPLOYRT$$

$$mecforce_{sr} = (labourforce_{sr} - unavailablelab_{sr})\, MECRATIO_{sr}$$

$$reglabourest_{sr,dr} = \begin{cases} mecforce_{sr} - MECTRANSOUT_{sr} & \text{for} \quad dr = sr \\ MECTRANSOUT_{sr} & \text{for} \quad dr \neq sr \end{cases}$$

3.8 Capital module

As already explained, two forms of capital are recognised in the model, built capital and natural capital. Within the capital module, natural capital is further separated into three types: agricultural land, coal, and oil/natural gas. The capital module keeps track of capital stocks, computes factor prices and quantities for capital, and redistributes the income from capital to economic agents. The full set of equations are available in Appendix Section B.8.

Capital Stocks Stocks of built capital held by each industry, $\mathbf{Builtcapital}_{dr,i}$, grow from the addition of new capital items and decline as a result of depreciation (Eq. B.178):

$$\frac{d}{dt}\left(\mathbf{Builtcapital}_{dr,i}\right) = netcapitalchange_{dr,i} + newcapital_{dr,i} - depreciation_{dr,i}$$

Depreciation on capital is calculated simply be assuming a constant depreciation rate, $RDEP_{dr,i}$, although one-off or scenario-specific adjustments to the depreciation rate are also available via the exogenous parameter $DEPSHFT_{dr,i}$ (Eq. B.206):

$$depreciation_{dr,i} = \mathbf{Builtcapital}_{dr,i}\left[RDEP_{dr,i}\,(1 + DEPSHFT_{dr,i})\right]$$

To ensure that the quantities of capital held by each industry align with the size of each industry as calculated under the primary module, the term $netcapitalchange_{dr,i}$ is also included in the calculation of the built capital stocks (Eq. B.222):

$$netcapitalchange_{dr,i} = \begin{cases} netratelandusechange_{IOag \to i}\, \mathbf{Builtcapital}_{dr,i} & \text{if} \quad i = \text{agricultural industry} \\ 0 & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

Total investment in new capital is determined under the investment and savings module through the calculation of aggregate investment value, $aggregateinvestv_{dr}$. In order to remove the effect of commodity price changes in determining the relative quantity of new capital items added to capital stocks each period, the investment value is divided by the current composite capital price, $\mathbf{Pinvestcc}_{dr}$, as determined under the savings and investment module, and further adjusted for exogenous set quantities of investment defined by $SETINVESTCQ_{dr,c}$.

Having determined the total quantity of new capital items, the next task is to determine how this new capital is distributed amongst industries. An extreme specification of the model would allocate investment to industries simply according to each industry's share of total capital income. These shares are defined by the auxiliary $capincomesh_{dr,i}$ defined as follows (Eqs. B.203, B.204, B.205):

$$capincomesh_{dr,i} = \frac{indcapincome_{dr,i}}{\sum_i (indcapincome_{dr,i})}$$

$$indcapincome_{dr,i} = builtuse_{dr,i}\, \mathbf{Pbuiltcap}_{dr,i}$$

$$builtuse_{dr,i} = \min\left[\mathbf{Builtcapital}_{dr,i}\, KSFCONVERT_{dr,i}\,,\, capitaltyped_{cap=BuilC,dr,i}\right]$$

where $\mathbf{Pbuiltcap}_{dr,i}$ is the industry specific built capital factor price, and $\mathbf{Builtcapital}_{dr,i} \times KSFCONVERT_{dr,i}$ specifies the annual quantity of capital factors provided.

Recognising however that some investment is mobile, only a proportion of new capital, specified by one minus the exogenous parameter $MOBILESH_{dr,i}$ is allocated to industries according to each industry's relevant share of capital income. Altogether the new capital allocated to each industry by this approach, termed $immobileinvest_{dr,i}$, is calculated as follows (Eq. B.198):

$$immobileinvest_{dr,i} = \left[\frac{aggregateinvestv1_{dr}\,(1 - NONPRODINVESTSH(t))}{\mathbf{Pinvestcc}_{dr}} + \sum_c (SETINVESTCQ_{dr,c})\right] \times (1 - MOBILESH_{dr,i})\, capincomesh_{dr,i}$$

The exogenous time series, $NONPRODINVESTSH$, has default values of zero but is included in the above equation to allow for scenarios where investments funds are allocated to the building of capital that does not in itself increase production (e.g. it may be enhancing environmental performance but not increasing industrial output).

The remaining quantity of new capital provided by investment is allocated to industries based on relative capital returns. Industries with above-average capital returns receive a larger share of this mobile investment capital than their share in capital income, while the converse occurs for industries from which capital returns are below-average. The following set of equations determines each industry's share of mobile investment capital, $mobileinvestsh_{dr,i}$ (Eq. B.199, B.200, B.201, B.202):

$$mobileinvestsh_{dr,i} = \frac{mobileinvest1_{dr,i}}{\sum_i (mobileinvest1_{dr,i})} ALLOCATESH_{dr} + INVESTCONSTSH_{dr,i}$$

$$mobileinvest1_{dr,i} = (INVESTPARAM_{dr,i}\, netreturn_{dr,i})^{EINVEST_{dr,i}} \times capincomesh_{dr,i}$$

$$netreturn_{dr,i} = grossreturn_{dr,i} - RDEP_{dr,i}$$

$$grossreturn_{dr,i} = \frac{\mathbf{Pbuiltcap}_{dr,i}\, KSFCONVERT_{dr,i}}{\mathbf{Pinvestcc}_{dr}}$$

For the above equations, $ALLOCATESH_{dr}$ is the share of mobile investment that is allocated to industries based on the relative returns to capital in those industries. $INVESTCONSTSH_{dr,i}$ is the industry share of regional investment held constant. $EINVEST_{dr,i}$ is a parameter that controls the degree to which investment allocated to industries response to changes in the rate of return on capital and $INVESTPARAM_{dr,i}$ is a parameter for scaling net return on capital.

Once the shares of mobile investment capital are determined, it is then possible to calculate mobile investment capital allocated to each industry, as well as total investment capital allocated to each industry (Eq. B.197, B.196):

$$mobileinvest_{dr,i} = \left[\frac{aggregateinvestv1_{dr}\,(1 - NONPRODINVESTSH(t))}{\mathbf{Pinvestcc}_{dr}} + \sum_{c}(SETINVESTCQ_{dr,c})\right] \times MOBILESH_{dr,i}\, mobileinvestsh_{dr,i}$$

$$newcapital_{dr,i} = mobileinvest_{dr,i} + immobileinvest_{dr,i}$$

To complete this section, we note that stocks of natural capital are held constant in the model, except for agricultural land (in rest of New Zealand) which is set to decrease at the historically observed rate of -0.72% per annum (Eq. B.179).

$$\frac{d}{dt}(\mathbf{Naturalcapital}_{dr,nct}) = CONVERSIONRT_{dr,nct}\, \mathbf{Naturalcapital}_{dr,nct}$$

For the Southland region, quantities of agricultural land supply are sourced from the primary module (Eq. B.223):

$$agrilandsup_{dr,i,nct} = \begin{cases} industryland_{IOag \to i} & \text{if} \quad nct = \text{Land1} \\ 0 & \text{if} \quad nct \neq \text{Land1} \end{cases}$$

Capital Factor Prices and Quantities As with other base prices in the model, the base prices for built capital and natural capital are determined by considering the balance of supply and demand (Eqs. B.180, B.181):

$$\frac{d}{dt}\left(\mathbf{Pbuiltcap}_{dr,i}\right) = \left(\left(\frac{1}{builtratio_{dr,i}}\right)^{\alpha^{pbuiltcap}} - 1\right) \mathbf{Pbuiltcap}_{dr,i}$$

$$\frac{d}{dt}\left(\mathbf{Pnaturalcap}_{dr,i,nct}\right) = \left(\left(\frac{1}{naturalcapratio_{dr,i,nct}}\right)^{\alpha^{pnatcap}} - 1\right) \mathbf{Pnaturalcap}_{dr,i,nct}$$

where the ratios of supply and demand are (Eqs. B.185, B.210):

$$builtratio_{dr,i} = \frac{builts_{dr,i}}{capitaltyped_{cap=BuilC,dr,i}}$$

$$naturalcapratio_{dr,i,nct} = \frac{indnaturalcaps_{dr,i,nct}}{naturalcaptyped_{dr,i,nct}}$$

A tree diagram showing how the supply ($builts_{dr,i}$ and $indnaturalcaps_{dr,i,nct}$) and demand ($capitaltyped_{cap=BuilC,dr,i}$ and $naturalcaptyped_{dr,i,nct}$) for capital factors is calculated is shown in Figure 3.7.

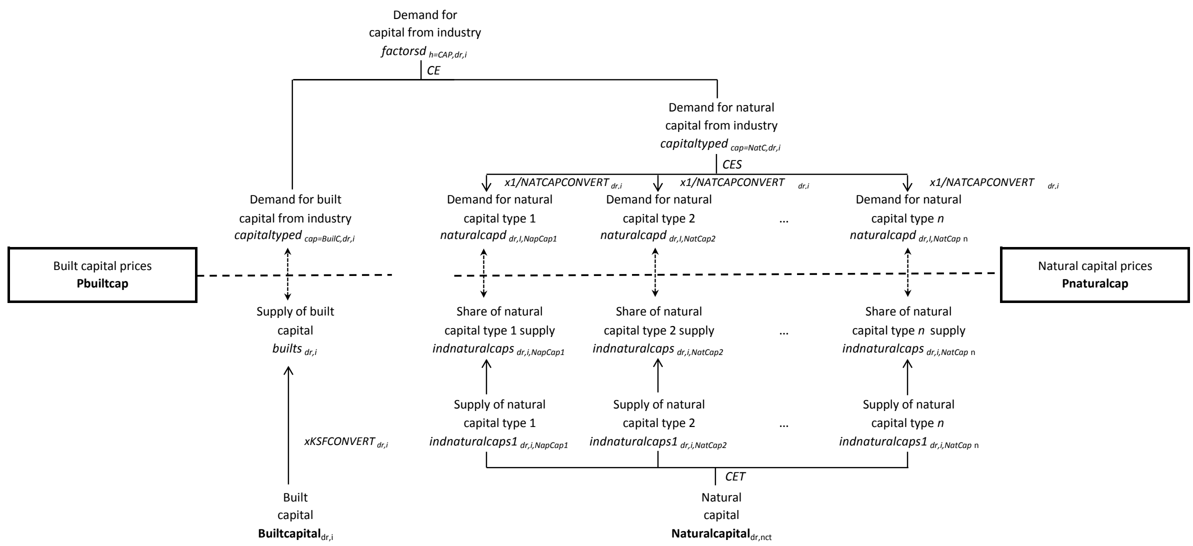

Figure 3.7 Tree diagram showing how the prices of built and natural capital are calculated from the supply and demand.

The demand equations pick up directly from the demand components of the factors module. Once again CES functions are used, first to split industry-specific composite capital demands, $factorsd_{h=cap,dr,i}$, into demands for built capital, $capitaltyped_{cap=BuilC,dr,i}$, and composite natural capital, $capitaltyped_{cap=NatC,dr,i}$, by the following equations (Eq. B.208)

$$capitaltyped_{cap=BuilC,dr,i} = \left[(scalecc1_{dr,i})^{\eta^{cc}_{dr,i}} \, sharecc1_{cap=BuilC,dr,i} \frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pbuiltcap}_{dr,i}} \right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \times factorsd_{h=CAP,dr,i}$$

$$capitaltyped_{cap=NatC,dr,i} = \left[(scalecc1_{dr,i})^{\eta^{cc}_{dr,i}} \, sharecc1_{cap=NatC,dr,i} \frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pcompnaturalcapd}_{cap=NatC,dr,i}} \right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \times factorsd_{h=CAP,dr,i}$$

And similar to other industry CES functions in the model, share and scale parameters for non-primary industries are sourced exogenously, while primary industry share and scale parameters come from the primary module (Eqs. B.221, B.220):

$$scalecc1_{dr,i} = \begin{cases} fiscalecapitalinput_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{cc}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

$$sharecc1_{cap,dr,i} = \begin{cases} fisharecapitalinput_{IOag \to i,cap} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{cc}_{cap,dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases}$$

The key difference between the two CES-based equations for built and natural capital is that demands for built capital are calculated by comparing the composite capital factor demand price, $\mathbf{Pfact}_{h=CAP,dr,i}$, with the price of built capital, $\mathbf{Pbuiltcap}_{dr,i}$, while demands for composite natural capital are calculated by comparing the composite capital factor demand price with the composite natural capital demand price, $\mathbf{Pcompnaturalcapd}_{dr,i}$. The price of built capital is a base price and thus is computed by comparing the supply of built capital to the demand for built capital (see Eqs. B.180, B.185, and B.186), while the composite natural capital demand price is computed like other CES composite demand prices in the model (see Eqs. B.182, B.215, and B.216).

To complete the demand side, composite natural capital demands are then disaggregated into demands for individual natural capital types using once again the CES approach (Eq. B.207):

$$naturalcapd_{dr,i,nct} = \left[(\gamma^{natcap}_{dr,i})^{\eta^{natcap}_{dr,i}} \delta^{natcap}_{dr,i,nct} \frac{\mathbf{Pcompnaturalcapd}_{dr,i}}{\mathbf{Pnaturalcap}_{dr,i,nct}} \right]^{\frac{1}{1-\eta^{natcap}_{dr,i}}} \frac{capitaltyped_{cap=NatC,dr,i}}{NATCAPCONVERT_{dr,i}}$$

Note that because the model permits natural capital to be measured according to a variety of units (e.g. hectares for land), the equation incorporates an exogenous conversion factor, $NATCAPCONVERT_{dr,i}$, that enables conversion to the selected unit of measurement for each natural capital type.

Turning now to the supply side, we have already noted that the capital module computes stocks of natural capital by type and industry-specific stocks of built capital. The supply of natural capital is transformed into individual industry allocations of natural capital, $indnaturalcaps_{dr,i.nct}$, based on a CET function (Eqs. B.213, B.211):

$$indnaturalcaps1_{dr,i,nct} = \left[(\theta^{natcap}_{dr,nct})^{\phi^{natcap}_{dr,nct}} \xi^{natcap}_{dr,i,nct} \frac{\mathbf{Pcompnaturalcaps}_{dr,nct}}{\mathbf{Pnaturalcap}_{dr,i,nct}} \right]^{\frac{1}{1-\phi^{natcap}_{dr,nct}}} \mathbf{Naturalcapital}_{dr,nct}$$

$$indnaturalcaps_{dr,i,nct} = \frac{indnaturalcaps1_{dr,i,nct}}{\sum_i (indnaturalcaps1_{dr,i,nct})} \mathbf{Naturalcapital}_{dr,nct} + agrilandsup_{dr,i,nct}$$

We then have available both the supply- and demand-side components of natural capital, allowing for computation of the natural capital base price, $\mathbf{Pnaturalcap}_{dr,i,nct}$ (see Eqs. B.181 and B.210).

To complete the supply components so that appropriate inputs can be generated for the factors module, the industry-specific endowments of natural capital of different types are also combined using a CES function to determine the total supply of composite natural capital to industries (see Eqs. B.188 and B.189). Finally, the supply of composite capital is determined by combining built capital supply with composite natural capital supply, also using the CES function (Eq. B.187) as shown in Figure 3.8. As part of this process, the price of composite natural capital supply, $\mathbf{Pcompnaturalcaps}_{dr,nct}$, is determined in the usual manner (Eqs. B.183 and B.212).

Capital Income and Expenditure Account A tree diagram that provides a summary of the different contributions to capital income and how capital is distributed is shown in Figure 3.9.

The principal source of income to the capital account is calculated simply by multiplying the quantity of capital factors used, as derived under the factors module, by the capital price, and

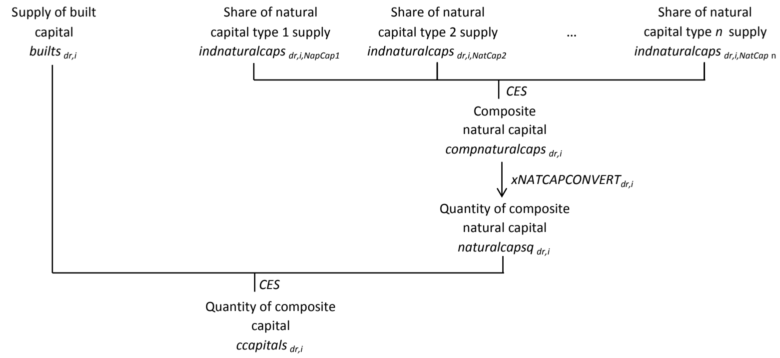

Figure 3.8 Tree diagram showing how composite capital supply is calculated from the supply of built and natural capital.

adjusting for any income that flows directly overseas. Also added to the capital income account is any net surplus in industry income, i.e. $\sum_i(\mathbf{Industrybalance}_{dr,i})$, as calculated under the industry module. Adjustments are also made to account for income needed to finance new municipal wastewater treatment schemes (loan payments and operating expenses), the responsibility of which is assigned to businesses (Eq. B.190, B.218, B.219):

$$\begin{aligned} capitalincome_{dr} =& \frac{\sum_i (factorsu_{h=CAP,dr,i}\, \mathbf{Pfact}_{h=CAP,dr,i})}{1 - RWFACTRT_{h=CAP,dr}} + capregtransout_{DReg1 \leftrightarrow DReg2} \\ &+ \sum_i (\mathbf{Industrybalance}_{dr,i}) - bussloanpayments_{dr} - \sum_c vmunopex_{dr,c,mf=Buss} \end{aligned}$$

$$\begin{aligned} vmunopex_{dr,c,mf} =& OPEXDEMANDBYTIME_{dr}(t) \times OPEXRESPBYTIME_{mf}(t) \\ &\times \mathbf{Pcompcommd}_{dr,c}\, WASTEMAP_c \end{aligned}$$

$$\begin{aligned} bussloanpayments_{dr=DReg1} &= \textstyle\sum_{lt} totalloanpayments_{rt=Normal,mf=Buss,lt} \\ bussloanpayments_{dr=DReg2} &= 0 \end{aligned}$$

The recognised capital income account, $\mathbf{Rcapincome}_{dr}$, adjusts to reflect the actual value of capital income (Eq. B.184):

$$\frac{d}{dt}(\mathbf{Rcapincome}_{dr}) = \frac{1}{\tau_{income}}(capitalincome_{dr} - \mathbf{Rcapincome}_{dr})$$

To ensure that this occurs quickly, the time for adjustment, τ_{income} is set equal to the time step. The capital income is then distributed according to fixed proportions derived from the base year SAM (Eqs. B.191 - B.195)

$$capregtransout_{dr} = \mathbf{Rcapincome}_{dr} \times CREGTRANSRT_{dr}$$

$$capentertrans_{dr} = \mathbf{Rcapincome}_{dr} \times CENTTRANSRT_{dr}$$

$$capgovttrans_{g,dr} = \mathbf{Rcapincome}_{dr} \times CGOVTTRANSRT_{g,dr}$$

$$caplocalhhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CHHLDTRANSRT_{dr}$$

$$capreghhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CRHTRANSRT_{dr}$$

where the proportions $CREGTRANSRT_{dr}$, $CENTTRANSRT_{dr}$, $CGOVTTRANSRT_{g,dr}$, $CHHLDTRANS$ and $CRHTRANSRT_{dr}$ sum to 1. The auxiliary, $capregtransout_{dr}$, is the value of capital income transferred to the capital account in the other NZ region. Whilst $capentertrans_{dr}$, $capgovttrans_{g,dr}$, $caplocalhhldtrans_{dr}$ and $capreghhldtrans_{dr}$ are respectively capital income transferred to enterprises, governments, within-region households and out-of-region households.

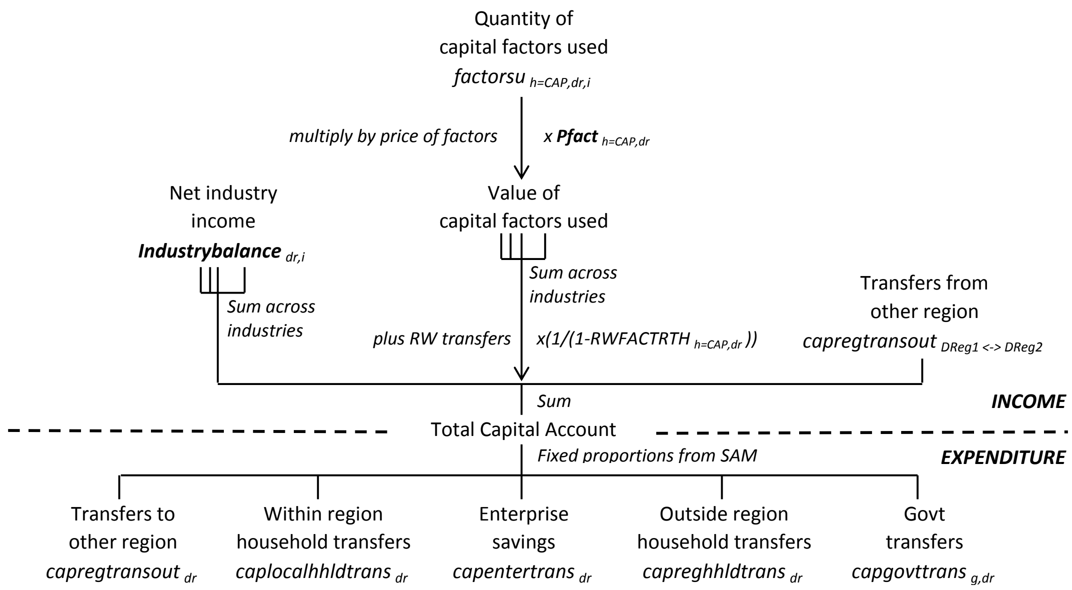

Figure 3.9 Tree diagram showing sources of capital and capital distribution. *SAM*: Social Accounting Matrix.

3.9 Investment & Savings module

We begin the description of this module by setting out the distinction between investment and savings. While in every day conversation the term investment might be used to refer to financial investments, such as shares and bonds, within the national accounts investment has quite a distinct and different meaning. In short, investment is the purchase of goods that will be used in the future to produce more goods and services. It therefore includes purchases of new houses, capital equipment, inventories and structures. Savings by contrast occur when an agent's income is greater than its expenditure, leading to a surplus of funds that are typically deposited in a bank. This module determines the values for both investment and savings, for each region. The full set of equations are available in Appendix Section B.9.

Savings Total savings for each region are comprised of savings which accrue specifically by regions, $regsavings_{dr}$, less interregional transfers of savings, $savregtransout_{dr}$, and savings derived from the rest of the world, $rwsavings_{dr}$ (Eq. B.237):

$$savingstotal_{dr} = rwsavings_{dr} + regsavings_{dr} - savregtransout_{dr}$$

A further adjustment is then required to savings that shares increases in financial intermediate services associated with new municipal wastewater treatment among the two regions in the model (Eq. B.238):

$$savingstotal1_{dr} = savingstotal_{dr} + \sum_{mf,lt} (totalloanpayments_{rt=FinIntServ,mf,lt}) \frac{savingstotal_{dr}}{\sum_{dr} savingstotal_{dr}}$$

A tree diagram that summarises the different contributions to the first estimate of overall savings is shown in Figure 3.10.

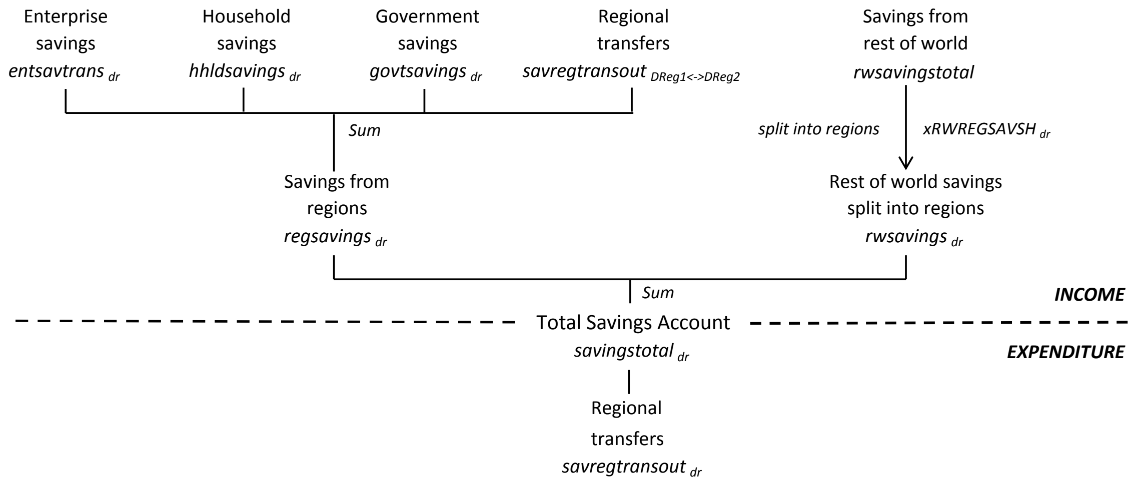

Figure 3.10 Tree diagram showing contributions to savings.

To determine the savings derived from the rest of the world within each region, the model assumes simply that total rest of world savings, $rwsavingstotal$, is shared among regions on a pro-rata basis according to relative contributions to total industry value added/GDP. The constant term, $RWREGSAVCONST_{dr}$, is also required in the equation, given that some regions start with negative rest of world savings in the base year (Eq. B.239):

$$rwsavings_{dr} = rwsavingstotal \frac{\mathbf{Holdregvaladd}_{dr}}{\sum_{dr} \mathbf{Holdregvaladd}_{dr}} + RWREGSAVCONST_{dr}$$

In turn, regionally-derived savings are the sum of enterprise, household, and government savings, i.e. $entsavtrans_{dr}$, $hhldsavings_{dr}$, and $govtsavings_{g,dr}$, respectively, plus the receipt of interregional transfers which are given by $savregtransout_{DReg1 \leftrightarrow DReg2}$ as transfers into $DReg2$ equal the transfers out of $DReg1$ ($savregtransout_{DReg1}$), and vice-versa. This gives the equation for regionally-derived savings (Eq. B.241):

$$regsavings_{dr} = entsavtrans_{dr} + hhldsavings_{dr} + \sum_{g} (govtsavings_{g,dr}) + savregtransout_{DReg1 \leftrightarrow DReg2}$$

Lastly, interregional transfers of savings are indexed to households within the origin region, where $SAVREGTRANSBS_{dr}$ defines the value of these transfers during the base year (Eq. B.242):

$$savregtransout_{dr} = SAVREGTRANSBS_{dr} \frac{\mathbf{Rhhldincome}_{dr}}{BASEHHLDACCOUNT_{dr}}$$

Total national rest of world savings, $rwsavingstotal$, is a relatively significant auxiliary within the model, deserving special consideration. We use a regression equation whereby total rest of world savings are found to be well predicted as a linear function of the national interest rate and the world GDP index, $WORLDGDPINDX(t)$, (Eq. B.239).

$$rwsavingstotal = \begin{cases} ACRWSAVINGS & \text{if} \quad Time < 4 \\ 100\,\mathbf{Interestrt} \times NZINTERESTWEIGHT & \\ +WORLDGDPINDX(t)\,GDPWEIGHT & \\ +RWSAVCONST + NETTRANSFERS(t) & \text{if} \quad Time \geq 4 \end{cases}$$

The rationale is that, the greater the interest rate achieved on saved funds in NZ, the more funds will be directed towards savings in NZ. Furthermore, simple growth in the world economy and hence the 'pool of funds' available will also increase rest of world savings. The relative weights given to the NZ interest rate and world GDP predictor variables are the exogenous parameters $NZINTERESTWEIGHT$ and $GDPWEIGHT$, while $RWSAVCONST$ is the regression constant.

A final complication is that in 2010/2011 (year 4 in this model) there were the devastating Canterbury earthquakes. This led to a significant drop in $rwsavings$, presumably due to net transfers related to asset loss and insurance effects. To account for this we have the exogenous $NETTRANSFERS(t)$ term for the historic time over which data is available (presently $t \leq 10$) from the Statistics NZ National Accounts.

The model also provides the option to override the regression equation for estimating rest of world savings with a time series of actual values, $ACRWSAVINGS(t)$, for the historic time over which data is available (presently $t \leq 10$ but we have chosen to use $t \leq 4$ as this provides a smoother transition in the early years of each model run). Note that since the model commences in 2006, the initial modelled years will cover the global financial crisis, a period during which savings behaviours were relatively unusual and hence not expected to be well represented by a generic regression equation based on historical data.

In NZ, market interest rates are influenced by the official cash rate, the interest rate set by the Reserve Bank of NZ. Importantly, changes in the official cash rate are the primary means by which the Reserve Bank maintains its core function of achieving and maintaining stability in the general level of prices. Increasing the interest rate can help to alleviate inflationary pressure via a number of routes. As already described under the households module, it is generally considered that household consumption is negatively correlated with interest rates, as households will need to devote a greater proportion of their budget towards repayment of mortgages when interest rates rise. A more subtle route is via the influence of interest rates on the exchange rate. Recall from the last paragraph that we model rest of world savings as dependant, in part, on the available interest rates in NZ. If there is an increased demand for NZ currency via rest of world savings this will help to appreciate the value of the NZ dollar relative to other currencies. In turn, by making NZ commodities relatively more expensive than foreign goods, this helps to reduce export demands and hence inflationary pressure. One further influence of the interest rate on commodity prices is described below in regards to investment demands.

Economist John Taylor proposed a policy rule whereby the short-term interest rate is adjusted to (1) movements of inflation from a desired value and (2) changes in the output gap (that is the difference between actual economic output ant its underlying trend) (Taylor, 1993). Although relatively simple, this rule and its variations has been widely regarded as a reasonable approximation of policy behaviour (cf. Bayoumi, 2004). For the first 3 years we use data to fix the interest rate to $ACTUALINTERESTRT(t)$. After that we follow a similar approach to model the change in the interest rate as adjusting to meet a variable termed the Taylor interest rate, $taylorinterestrt$, with the period of adjustment controlled by the constant $\tau_{interest}$ (Eq. B.225):

$$\frac{d}{dt}\left(\mathbf{Interestrt}\right) = \begin{cases} \frac{1}{\tau}\left(ACTUALINTERESTRT(t) - \mathbf{Interestrt}\right) & \text{for} \quad t < 3 \\ \frac{1}{\tau_{interest}}\left(taylorinterestrt - \mathbf{Interestrt}\right) & \text{for} \quad t \geq 3 \end{cases}$$

The Taylor interest rate, in turn, is determined by differences between (1) the inflation rate and the target inflation rate (assumed to be 2% per annum) and (2) the GDP gap. The latter is the percentage shortfall of GDP from an estimate of its natural rate, $NATURALGDP(t)$ (Eqs. B.245, B.229):

$$taylorinterestrt = \begin{cases} INTERESTCONST + INTERESTINFLW\left(desiredinflationrt - 0.02\right) \\ \quad + INTERESTGDPW\, ACTUALGDPGAP(t) & \text{for} \quad t \leq 3 \\ INTERESTCONSTGFC + INTERESTINFLW\left(\mathbf{Inflationrt} - 0.02\right) \\ \quad + INTERESTGDPW\, gdpgap & \text{for} \quad t > 3 \end{cases}$$

$$gdpgap = \frac{realgdp}{NATURALGDP(t)} - 1$$

The respective weights given to the inflation and GDP components of the calculation, $INTERESTINFLW$ and $INTERESTGDPW$, are set specific to the NZ context based on historic data.

Finally, we can note that once we have calculated the interest rate and inflation rates (the latter is available from the output reporting module) we can also determine the real interest rate (Eq. B.230):

$$realinterestrt = \mathbf{Interestrt} - \mathbf{Inflationrt}$$

Investment A tree diagram in Figure 3.11 shows how available investment funds are calculated, and where investment is allocated. For each region, the primary input to the funds available for investment is labelled 'variable investment'. Investment funds are assumed to be positively correlated with changes in the total value of regional savings. This helps to capture general trends in the size of the economy but also reflects that with more savings, banks will have more funds available to loan for investment spending. At the same time, it is also assumed that the funds allocated to investment are negatively correlated with the real interest rate. This is because the opportunity costs of borrowing money to invest in capital goods increases when the real interest rate increases, and people will also be spending more on repaying existing loans and mortgages so will have less available to invest. These relationships are integrated into the model by the equation for $aggregateinvestv_{dr}$ (Eq. B.235):

$$aggregateinvestv_{dr} = \left[\left(realinterestrt\, ALPHA + \sum_{dr}\left(savingstotal_{dr}\, BETA\right) + INVESTCONST\right)\frac{\mathbf{Holdregvaladd}_{dr}}{\sum_{dr}\mathbf{Holdregvaladd}_{dr}} + REGINVESTCONST_{dr}\right] \times \left(1 - INVESTINDIRECTTAXRT_{dr}\right)$$

where $ALPHA_{dr}$, $BETA_{dr}$, and $INVESTCONST_{dr}$ are respectively the weights given to the interest rate, savings, and the regression constant. The fraction term in the equation shares total investment calculated at the national level among regions based on their relative total value added/GDP. The equation also includes an adjustment for the proportion of investment funds within each region that are allocated towards payment of indirect taxes. This proportion, i.e. $INVESTINDIRECTTAXRT_{dr}$, is assumed to remain constant with the base year. The total value of indirect taxes on investment, investindirecttaxdr, is a necessary input for the government module and is calculated simply as follows (Eq. B.244):

$$investindirecttax_{dr} = aggregateinvestv1_{dr} \frac{INVESTINDIRECTTAXRT_{dr}}{1 - INVESTINDIRECTTAXRT_{dr}} + MUNCAPINVESTBYTIME_{dr} \times MUNINDTAXRT \frac{\textbf{Gdpindex}}{1000}$$

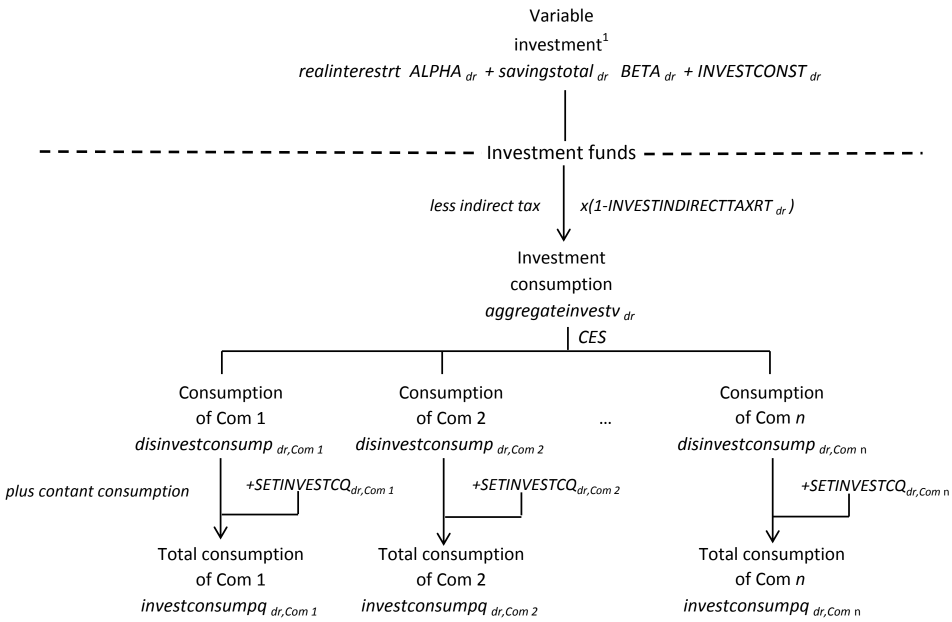

Figure 3.11 Tree diagram showing how available investment is calculated and where it is allocated. *CES*: Constant Elasticity of Substitution calculation.
[1] Determined empirically from a linear regression fit to data.

Once an initial calculation of regional aggregate investment is made, an adjustment is then required to account for funds that are compulsory allocated towards municipal wastewater treatment (see municipal module for specification of *investadjustcap* and *investadjustland*) (Eq. B.236):

$$aggregateinvestv1_{dr} = aggregateinvestv_{dr} - (investadjustcap + investadjustland) \times \frac{aggregateinvestv_{dr}}{\sum_{dr} aggregateinvestv_{dr}}$$

Once the funds available for investment are determined, the module operates in a similar manner to the household and government modules. That is the composite quantity of investment commodities consumed is calculated simply by dividing the aggregate investment value by the composite investment price, $\textbf{Pinvestcc}_{dr}$. This composite quantity is then disaggregated by a CES function so as to determine the individual commodity types consumed for investment,

$disinvestconsump_{dr,c}$ (Eq. B.234):

$$disinvestconsump_{dr,c} = \left[(\gamma_{dr}^{investc})^{\eta_{dr}^{investc}} \delta_{dr,c}^{investc} \frac{\mathbf{Pinvestcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}} \right]^{\frac{1}{1-\eta_{dr}^{investc}}} \frac{aggregateinvestv_{dr}}{\mathbf{Pinvestcc}_{dr}}$$

In this equation $\eta_{dr}^{investc}$ is the commodity substitution parameter for investment consumption, while $\gamma_{dr}^{investc}$ and $\delta_{dr,c}^{investc}$ are the CES scale and share parameters. Additionally, $\mathbf{Pcompcommd}_{dr,c}$ is the commodity-specific price by agents demanding commodities, as obtained from the commodities module.

Like other composite prices, the composite investment price is calculated via a series of three equations (Eqs. B.226, B.232, B.231):

$$\frac{d}{dt} (\mathbf{Pinvestcc}_{dr}) = \frac{1}{\tau_{prices}} (actualpinvestcc_{dr} - \mathbf{Pinvestcc}_{dr})$$

$$qinvestcc_{dr} = \gamma_{dr}^{investc} \left[\sum_{c} \left(\delta_{dr,c}^{investc} \left(disinvestconsump_{dr,c} \right)^{\eta_{dr}^{investc}} \right) \right]^{\frac{1}{\eta_{dr}^{investc}}}$$

$$actualpinvestcc_{dr} = \frac{\sum_c \left(disinvestconsump_{dr,c} \, \mathbf{Pcompcommd}_{dr,c} \right)}{qinvestcc_{dr}}$$

Once the composite investment demands have been disaggregated into demands for individual commodities a final adjustment is made to account for small quantities of goods that are assumed to be consumed each year and are set exogenously. This provides the final quantities of investment commodity consumption, $investconsumpq_{dr,c}$ (Eq. B.243):

$$investconsumpq_{dr,c} = disinvestconsump_{dr,c} + SETINVESTCQ_{dr,c} + muninvestconsump_{dr,c}$$

Presently, this adjustment is made simply because the base SAM records small quantities of negative investment consumption for certain commodities. These are all negligible quantities and probably relate to changes in stocks during the base year. See also the municipal module for explanation of $muninvestconsumpt$.

3.10 Municipal module

The municipal module is included within the Southland Economic Model so that scenarios can be tested involving introduction of new municipal wastewater treatment schemes. The municipal module is controlled by a separate policy interface (the Municipal Interface) currently programmed in Microsoft Excel. Informed by a number of case studies undertaken within Southland on wastewater treatment options, the Municipal Interface is used to set trajectories (or scenarios) for wastewater treatment upgrades, estimate the likely resources required for such upgrades, and allocate responsibility for funding of these upgrades. The devised scenarios are implemented in the Southland Economic Model through a set of exogenous parameters as described in Table A.21.

As it was desirable to limit the steps involved for non-technical experts in running scenarios, all exogenous parameters created for a scenarios are of the same type and apply for an entire model run (i.e. no variables are time dependent). However in reality, costs and expenditures associated with municipal treatment schemes will be highly variable over time. To enable the model to capture changes across time, it was therefore necessary to introduce a new time-related model subscript as a work-around (termed 'year mitigated', see Table A.15). Although it makes some of the calculations a little convoluted, the model is able to utilise the year mitigated subscript to determine costs and expenditures at different points in time.

As already explained in the savings and investment module, when calculating the commodities demanded for investment consumption, an adjustment termed $muninvestconsump_{dr,i}$ is made to account for goods and services required specifically for investment in new municipal treatment plants. This adjustment is determined by taking the total quantity of expenditure in base dollar terms on new municipal capital (excluding land) across time, termed $MUNCAPINVESTBYTIME_{dr}(t)$, [3] and multiplying by $MUNINVESTRATIO$ which defines the average ratio of a commodity demanded per total commodity demand for municipal investment (Eq. B.255):

$$muninvestconsump_{dr,c} = MUNCAPINVESTBYTIME_{dr}(t) \times MUNINVESTRATIO_c$$

The next components of the municipal module are concerned with calculating the total value of investment in municipal wastewater treatment, so that expenditure on paying loans that finance this investment can be estimated. The variable, *investadjustcap*, defines total value of investment on non-land capital and is calculated by multiplying the quantities of commodities consumed for municipal investment by the relevant prices of commodities and adjusting for taxes. The variable, *investadjustland*, similarly defines the total value of investment in land capital (Eqs. B.254, B.256):

$$\begin{aligned} investadjustcap = & \sum_{dr,c} \left(muninvestconsump_{dr,c}\, \mathbf{Pcompcommd}_{dr,c} \right) \\ & + \sum_{dr} \left(MUNCAPINVESTBYTIME_{dr}(t) \times MUNINDTAXRT\, \frac{\mathbf{Gdpindex}}{1000} \right) \end{aligned}$$

$$investadjustland = \sum_{dr} \left(MUNLANDINVESTBYTIME_{dr}(t)\, \frac{\mathbf{Gdpindex}}{1000} \right)$$

For the different loan values taken on each year, it is then possible to calculate the associated periodic and annual loan payments, *periodicloanpayments* and *annualloanpayments*, given information from the Municipal Interface on the loan interest rate, $LOANRATE$, and assuming a fixed number of payments made per year, $PAYMENTSPERYEAR$, and years over which the loan is taken out, $YEARSFORLOAN$ (Eqs. B.252, B.253, B.257, B.258):

$$annualloanpayments_{rt,lt} = periodicloanpayments_{rt,lt}\, PAYMENTSPERYEAR$$

$$periodicloanpayments_{rt,lt=Capital} = investadjustcap\, \frac{periodicrate_{rt}\,(1+periodicrate_{rt})^{numberpayments}}{\left((1+periodicrate_{rt})^{numberpayments}\right)-1}$$

$$periodicloanpayments_{rt,lt=Land} = investadjustland\, \frac{periodicrate_{rt}\,(1+periodicrate_{rt})^{numberpayments}}{\left((1+periodicrate_{rt})^{numberpayments}\right)-1}$$

[3] For reasons outlined in the preceding paragraph, $MUNCAPINVESTBYTIME$ as well as $MUNLANDINVESTBYTIME$ are actually defined in the Municipal Interface not as time varying inputs but instead by a subscript denoting the year the investment occurs. Multi-layered 'IF' functions are then used to translate this information into single variables that change across time. For simplicity this step in the calculations has been omitted in the equations

$$numberpayments = YEARSFORLOAN \times PAYMENTSPERYEAR$$

$$\begin{aligned} periodicrate_{rt=Normal} &= \frac{LOANRATE}{PAYMENTSPERYEAR} \\ periodicrate_{rt=FinIntServ} &= \frac{LOANRATE-FISSHARE}{PAYMENTSPERYEAR} \end{aligned}$$

Note that in the above equations, loan payments are calculated twice for that which is attributed towards payment of financial intermediate services (denoted by rate type subscript $FinIntServ$) and that which is the normal loan payment (denoted by rate type subscript $Normal$). This occurs through an assumption that a fixed part of the loan interest rate, $FISSHARE$, is devoted towards payments for financial intermediate services. Keeping the separation between loan payments for financial intermediate services and other loan payments is necessary for the eventual calculation of demands for financial intermediate services, a type of commodity demand (see Eq. B.247).

Over the whole of Southland Region it is possible that there will be multiple different loans taken out, each with different payment values, implementation dates, and end dates. The next components of the municipal module calculate the total loan payments that need to be made annual, across all loans. The module also assigns responsibility for loan payments to one of three agents, i.e. households (subscript $Hhld$), central government (subscript $Cgovt$) and businesses (subscript $Buss$). Note that it has been recognised that local government may also take on responsibility for financing municipal capital investments and hence take on responsible for loan payments. However, since this agent is in turn likely to finance such expenditure through increased rate take from households, such an arrangement is considered included within the category of household responsibility. Altogether total loan payments faced over time by funding agents, $totalloanpayments_{rt,mf,lt}$, is determined as follows (Eqs. B.248, B.249, B.246, B.250, B.250, B.259):

$$totalloanpayments_{rt,mf,lt} = \sum_{yrmt} loanpayments_{yrmt,rt,mf,lt}$$

$$loanpayments_{yrmt,rt,mf,lt} = \begin{cases} maxloanpayment_{yrmt,rt,mf,lt} & \text{if} \quad \mathbf{Loantopay}_{yrmt,rt,mf,lt} > 0 \\ 0 & \text{if} \quad \mathbf{Loantopay}_{yrmt,rt,mf,lt} \leq 0 \end{cases}$$

$$\frac{d}{dt}(\mathbf{Loantopay}_{yrmt,rt,mf,lt}) = totaltopay_{yrmt,rt,mf,lt} - loanpayments_{yrmt,rt,mt,lt}$$

$$totaltopay_{yrmt,rt,mf,lt} = loanpaymentsbyyearloan_{yrmt,rt,mf,lt}\,YEARSFORLOAN$$

$$loanpaymentsbyyearloan_{yrmt,rt,mf,lt} = \begin{cases} annualloanpayments_{rt,lt}\,CAPINVESTRESP_{yrmt,mf,lt} \\ \qquad \text{if} \quad LOANYR - 0.5 < Time \leq LOANYR + 0.5 \\ 0 \qquad \text{if} \quad LOANYR - 0.5 \geq Time > LOANYR + 0.5 \end{cases}$$

$$maxloanpayment_{yrmt,rt,mf,lt}(t+1) = \begin{cases} loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t+1) \\ \qquad \text{if} \quad loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t+1) \\ \qquad\qquad \geq loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t) \\ loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t) \\ \qquad \text{if} \quad loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t+1) \\ \qquad\qquad < loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t) \end{cases}$$

3.11 Primary module

The primary module enables the Southland Economic Model to be used to test various scenarios involving changes to farming systems, as might occur as a result of water policy initiatives. Like the municipal module, the primary module is controlled by a separate policy interface programmed in Microsoft Excel, this time termed the 'Primary Interface'. Through this interface, users select values for all exogenous parameters defined in Table A.20.

The calculations contained within the primary module are targeted towards five general functions, i.e.: (1) calculation of land use changes over time, both in terms of changes in whole farm types (e.g. from drystock farming to dairy cattle farming) as well as changes in the way land is allocated to different types of farming or mitigation systems (e.g. from a drystock farm operating according to its current system and practices, to the same drystock farm with a crop mitigation option implemented); (2) calculation of the level of de-intensification of agricultural systems as a result of land retirement, (3) calculation of the perceived or expected returns per hectare for different land use options, as this is a necessary input for (1); (4) calculation of the costs faced across time for different agents as result of new fencing and/or farm plan requirements, and (5) calculation of industry-level changes in input and output parameters so that this information can feed to other components of the model. Each of these components of the module are discussed below.

For the operation of the primary module, it was necessary to introduce several additional sets of subscripts to the model, as set out in Table 2.1 and Appendix A. Importantly, within the primary module, the Southland region is divided into 33 different economic zones (subscript ez). These economic zones are defined by six Freshwater Management Unit categories, as well as seven unique categories defined by soil/rainfall/slope/farm size.[4] Financial data on the operation of farms/forestry, and the commodities used and produced by farms/forestry is obtained through the 57 different case study 'farms' modelled (subscript $mofa$). Such data is arrayed not only according to the particular farm modelled, but also the mitigation state (subscript mt) modelled for each farm, with up to nine different mitigation states possible.

An obstacle that needed to be overcome in the construction of the primary module, was to reduce the quantity of data held and calculations undertaken in the model, so that run times would be reduced. With 57 different case study farms, 9 potential mitigation states, and 33 different economic zones, the model would have needed to keep track of around 17,000 different combinations. To reduce this complexity an additional subscript was introduced, termed Farm Type (subscript ft) which essentially allows for only 17 different case study farms to be matched with any particular economic zone. This is possible because while there are 44 different modelled drystock case study farms, a maximum of eight sheep and beef farms and 5 deer farms are applied to any one zone. Similarly, although there are ten different modelled dairy farms, only one is used for any particular economic zone. Careful mapping between case study farms and farm types is however required in the model and its input data sets to successfully implement this work-around.

A further subscript type introduced in the primary module is IO agricultural industries (subscript $IOag$). This is a subset of all industries included in the general components of the model and includes only those three industries that are primary. However, given that the first industry, $AGIn01$, is quite aggregate and includes farming systems for which there are differently assigned case study farms, a further subscript category is introduced termed agricultural industries (sub-

[4]Although this would in theory allow for up to 42 different economic zones, only 33 are recognised in the model because some of the potential categories did not exist in Southland, or the land areas occupied were sufficiently small to warrant exclusion from the model.

script *agind*). Under this category $AGIn01$ is split into four new industries, i.e. sheep and beef, deer, arable, and horticulture.

Land Use Change The model keeps track of land use through the stock **Landuse** which is subscripted for economic zone, farm type, and mitigation state. All land use is measured according to effective hectares. Even without the implementation of freshwater policies, land use is likely to change over time as land owners respond to changing preferences and system conditions, including changes in financial returns for different land use options. The stock $\mathbf{Landuse}_{ez,ft,mt}$ thus receives inputs of new land acquired via land use change, $landusechangein_{ez,ft,mt}$, and outputs of land lost to land use change $landusechangeout_{ez,ft,mt}$. Land use change occurring as a result of adoption of new mitigation states also creates inflows, $mitigationin_{ez,ft,mt}$, and outflows, $mitigationout_{ez,ft,mt}$, for the land use stock. Furthermore, land can be retired altogether from primary production, with this occurring through the outflow $farmretirement_{ez,ft,mt}$. In summary the equation for $Landuse_{ez,ft.mt}$ is defined as (Eq. B.263):

$$\begin{aligned}\frac{d}{dt}(\mathbf{Landuse}_{ez,ft,mt}) = & \, actuallanduse_{ez,ft,mt} + landusechangein_{ez,ft,mt} - landusechangeout_{ez,ft,mt} \\ & - farmretirement_{ez,ft,mt} + mitigationin_{ez,ft,mt} - mitigationout_{ez,ft,mt}\end{aligned}$$

The additional term used in this above equation, $actuallanduse_{ez,ft.mt}$, allows for users to cause the stock to match known trajectories of land use change, where this might be desirable (e.g. for part of each model run that covers the past and where data on historic land use change is available) - see Eq. B.264.

Turning now to the calculation of other other flows to/from **Landuse**, except in situations where we wish to force land use change to match a determined trajectory (i.e. $Time < KNOWLANDTIME$), the *landusechangeout* flow is calculated simply by multiplying the current stock value by a maximum rate of land use change, $MAXCHANGERT_{ft}$. Furthermore, since land is conserved, all land use change out of a particular land use must be balanced by land use change into another land use (Eqs. B.326, B.307, B.301):

$$landusechangeout_{ez,ft,mt} = \begin{cases} \mathbf{Landuse}_{ez,ft,mt} & \\ \quad \times MAXCHANGERT_{ft} & \text{if} \quad Time \geq KNOWLANDTIME \quad \text{and} \\ & \qquad Time \leq HOLDLANDTIME \\ 0 & \text{if} \quad Time < KNOWLANDTIME \quad \text{or} \\ & \qquad Time > HOLDLANDTIME \end{cases}$$

$$landusechangein_{ez,ft,mt} = \begin{cases} \sum_{agi}(landtoreallocate_{ez}\; agindshareofallocation_{ez,agi} & \\ \quad \times fsshareallocation_{ez,ft,mt,agi}) & \text{if} \quad Time \geq KNOWLANDTIME \\ \qquad 0 & \text{if} \quad Time < KNOWLANDTIME \end{cases}$$

$$landtoreallocate_{ez} = \sum_{ft,mt} landusechangeout_{ez,ft,mt} + \sum_{ft} landnomitigation_{ez,ft}$$

Note that in the last equation, the land allocated to land use change includes not only the quantity of land changing out of land uses, but also the sum of $landnomitigation_{ex,ft,mt}$. The inclusion of this term is to cover land that is required to change out of a mitigation state, but there is no other available mitigation state that is permitted within the same farm type so more general land use change must occur - see Eq. B.300.

In the calculation of $landusechangein$, the proportion of land within a given economic zone that is allocated to a particular farm type and mitigation is based on the product of two separate shares. The first, $agindshareofallocation_{ez,agri}$, defines the proportion of land within a zone allocated to a particular agricultural industry while the second, $fsshareallocation_{ez,ft,mt,agri}$, defines for a given economic zone and agricultural industry, the proportion allocated to a particular farm type and mitigation state. The full set of equations required to calculate the first share are set out in Appendix B (see Eqs. B.292, B.293, B.294, B.296, B.297, B.298, B.303). In summary: (1) a CET function is used to share land, subject to land use change among agricultural industries based on each industry's average perceived returns per hectare (taking into consideration all farm types and mitigation states belonging to an industry) and the assigned elasticity of transformation among land uses; (2) The parameter values used in the CET function are set through calibration; and (3) When comparing returns per hectare for agricultural industries, mitigation states that are no longer permitted to continue are carefully excluded. The second share is based simply on the current distribution of farm types and mitigation states within an economic zone and farm type, excluding any mitigation states that are no longer permitted to occur - see Eq. B.299.

In terms of the mitigation based out-flow from the **Landuse** stock, i.e $mitigationout$, a separate Appendix (Appendix C) is included to explain how the flow is derived. As a summary, our starting point is that if a mitigation state is no longer allowed as a result of an implemented policy (say just for example that all dairy cattle farming is no longer permitted to occur in the current i.e. baseline state, and instead all farms are required to achieve at least 10 percent nitrogen loss reduction), then it is unlikely that all farms and associated land use would switch to a new mitigation state at the same point in time. Instead, we are more-likely to see a phase-in period over which new mitigation state or states are progressively adopted across the region's dairy cattle farming land. The model assumes a typical s-shaped adoption profile. However, through the Primary Interface, users can choose to speed up or slow down the overall pattern of adoptions. Furthermore, users set the points in time when adoption out of a mitigation state first commences (for example, could be set to correspond to the date when it becomes certain that a new rule will soon apply) and when all adoption must be complete (often the date when a new rule takes effect). A further nuance of the model is that it allows for scenarios where only some land in an economic zone and farm type must adopt out of a mitigation state. This is implemented through a maximum allowable land use control in the Primary Interface, and it is only hectares of land above this control that will be required to adopt new mitigations. Altogether, Eqs B.311, B.322, B.323, B.324, B.327, B.328, B.329, B.330, B.331, and B.332, as set out in Appendix B, implement this component of the model.

Once again, because of the principle of conservation of land, the total land lost from an economic zone as a result of mitigation adoptions should be balanced by inflows of new land in that same economic zone. For this component of the model, it is assumed that landowners will choose to adopt the next most profitable permitted mitigation state that belongs to the same farm type as their original land use. As already explained above, if there is no available mitigation state in the same farm type that is permitted for adoption then the relevant hectares of land falls to the land available for general land use change - in other words a new farm type within the same agricultural industry, or even a new agricultural industry type, can be adopted. Eqs. B.290, B.291, B.309, B.317, B.320, B.321, and B.325, in Appendix B, implement this component of the model.

The final flow from the **Landuse** stock is that associated with land retirement, $farmretirement_{ez,ft,mt}$. In the Primary Interface, for each economic zone, users specify farm retirement over time at the scale of agricultural industry. In the model it is then assumed that this retirement is split among farm types and mitigation states on a pro-rata basis, according to current distributions of farm types and mitigation states within an economic zone, see Eqs. B.305 and B.310. Given that the model calculates primary industry commodity production (outputs) and demand for intermediate inputs and factors (inputs) by multiplying per hectare supply/demand ratios by land use, removing land out of the **Landuse** stock immediately results in a downsizing of the associated industries.

To complete this section, we note that there are also some aggregate categories of land use and land use change calculated within the primary module as this information is required within other modules - see Eqs. B.304, B.306, B.349, and B.350.

De-intensification In addition to the land retirement option discussed in the preceding paragraph, the Primary Interface recognises the possibility for land retirement that results in a de-intensification of selected land uses. This option is most likely to be utilised when investigating scenarios that involve land retirement within farms to other uses. For example, planting buffer strips along streams, setting aside land for wetlands and/or planting critical source areas in forest. Unlike the retirement option discussed above which assumes that all inputs and outputs to farm activities will scale down in proportion to quantities of effective hectares retired, under this form of land use retirement it is assumed that only outputs and intermediate inputs scale down - i.e. payments for depreciation and labour per hectare remain unchanged. The model also recognises two sub-forms of de-intensification retirement, one relating to retirement to non-agricultural productive uses and one relating to retirement to farm forestry. The latter is associated with production of alternative economic commodities while the former is not. The calculations for this component of the model can be found in Eqs. B.260, B.261, B.278, B.279, B.280, in Appendix B.

To further complicate matters, a special type of mitigation (subscript *miti09*) was added to the Primary Interface which allows for the option of testing land de-intensification as a means of achieving specific nitrogen load targets. The outcome of the mitigation is the same as that discussed in the last paragraph, but operates differently in terms of user inputs. In the above case users directly enter the quantity of land retired, while in the case of the mitigation option, users enter the required N loss per effective hectare and the model determines the necessary quantity of land retired. Eqs. B.284, B.286, B.360, and B.361 deal with this specific aspect of the model.

Expected returns to land The principal objective of this component of the primary module is to calculate the per hectare returns to land at the same scale of resolution as the **Landuse** stock. As mentioned above, this information is a necessary input to the calculations for land use change. More precisely, the model calculates the *perceived* returns to land for each economic zone, farm type and mitigation state, i.e. stock $\mathbf{Percfsreturnsperha}_{ez,ft,mt}$. While based on actual returns per hectare, $fsretunsperha_{ez,ft,mt}$, the stock adjusts over time to both smooth out short term fluctuations and account for potential lags between actual changes and accepted/perceived changes (Eq. B.262):

$$\frac{d}{dt}(\mathbf{Percfsreturnsperha}_{ez,ft,mt}) = \frac{fsreturnsperha_{ez,ft,mt} - \mathbf{Percfsreturnsperha}_{ez,ft,mt}}{ADJUSTTIME}$$

Conceptually, the calculation of returns to land is straightforward. On a per hectare basis, it is the net difference between income from sale of goods/services produced on farms and the costs of production. The latter includes purchases of intermediate inputs, payments for labour and depreciation, and income allocated as a payment/rent of built capital. Nevertheless, there are several steps and calculations required in the model because: (1) much of the data and calculations undertaken elsewhere in the model are undertaken in nominal terms but to calculate the value of returns to land we need to work in real terms, which requires adjusting for current prices, (2) base data on inputs and outputs for farm types and mitigation states enters the model subscripted according to case study farm which must then be mapped to farm types for each economic zone, (3) the general components of the model allow for background productivity change over time within economic industries - this can also be included when calculating income and expenses for primary activities, and (4) adjustments need to be made to account for de-intensification of agriculture, which may include switching some land to farm forestry. See Eqs. B.289, B.266, B.267, B.268, B.285, B.287, B.357, B.358, and B.359 for the full calculations.

Fencing and farm plans Within the Primary Interface, users have the option of testing scenarios that include requirements for additional fencing on farms and/or implementation of farm plans. This component of the primary module simply translates the inputs from the interface into a time series of costs faced by each agricultural industry, and also determines the demand for fencing contract services (in the case of fencing), and farm advisor/ farm plan creation services as well as council processing services (in the case of farm plans). See Eqs. B.281, B.282, B.283, B.312, B.313, B.314, B.315, B.316, B.318, B.319, B.344, B.345, B.346, and B.347, in Appendix B for this component of the model.

Industry-level inputs and outputs Due to adoption of mitigations, land use change, and land retirement/de-intensification occurring within the primary module, the structure of each primary industry at an aggregate level will potentially be changing over time. The final components of the module therefore deal with updating industry-level variables used elsewhere in the model. As explained in the industries, commodities and factors modules, the calculation of industry production and consumption in the Southland Economic Model is based around sets of nested CES/CET functions. For each time step in the model, it is therefore necessary to update the parameters used in these functions for the primary industries to reflect changes occurring with the primary module.

Eqs. B.143, B.288, B.336, and B.348 calculate the updated primary industry commodity supply scale and and share parameters (i.e. to replace $\phi^{comsup}_{sr,i}$ and $\xi^{comsup}_{sr,i,c}$ respectively). Also calculated, is the relative rate of commodity production compared to the base year (Eq. B.351).

Eqs. B.265, B.275, B.276, B.333, B.337, and B.341 are concerned with calculating the revised share and scale parameters for factors/composite intermediate inputs (replacing $\gamma^{fi}_{dr,i}$ and $\delta^{fi}_{input,dr,i}$). Similarly, Eqs. B.277, B.335, and B.340 calculate revised share and scale parameters for each type of commodity input (replacing $\gamma^{cominput}_{dr,i}$ and $\delta^{cominput}_{dr,i,c}$). Finally, Eqs. B.269, B.270, B.271, B.272, B.273, B.274, B.334, B.338, B.339, B.342, and B.343 determine the scale and share parameters for capital/labour inputs (replacing $\gamma^{fact}_{dr,i}$ and $\delta^{fact}_{h,dr,i}$) and the scale and share parameters but for built capital/land inputs (replacing γ^{cc}_{dr} and δ^{cc}_{dr}).

3.12 Rest of world module

The rest of world module tracks flows of funds into and out of the NZ economic system. The full set of equations are available in Appendix Section B.12.

A primary task of the rest of world module is to determine changes in the exchange rate, **Exchangert**. The exchange rate can be conceptualised as the quantity of US dollars per NZ dollar, but rescaled so that the rate is equal to one for the base year. Importantly, the model treats the exchange rate in an analogous manner to base prices in other modules. That is, the exchange rate fluctuates in response to differences in supply and demand quantities following the equation (Eq. B.362):

$$\frac{d}{dt}(\mathbf{Exchangert}) = \begin{cases} \frac{1}{TIME\ STEP}\left(ACTUALEXCHANGERT - \mathbf{Exchangert}\right) & \text{for} \quad t < 3 \\ \left(\left(\frac{1}{bopratio}\right)^{\alpha^{exchangert}} - 1\right)\mathbf{Exchangert} & \text{for} \quad t \geq 3 \end{cases}$$

with the rate of response dependant on the exogenous alpha parameter, $\alpha^{exchangert}$. In this case the supply and demand quantities compared are the supply and demand for NZ currency, also respectively termed rest of world expenditure and rest of world income. Also, the ratio of rest of world income to rest of world expenditure is termed the balance of payments ratio, *bopratio* (Eq. B.363):

$$bopratio = \frac{rwincome}{rwexpenditure}$$

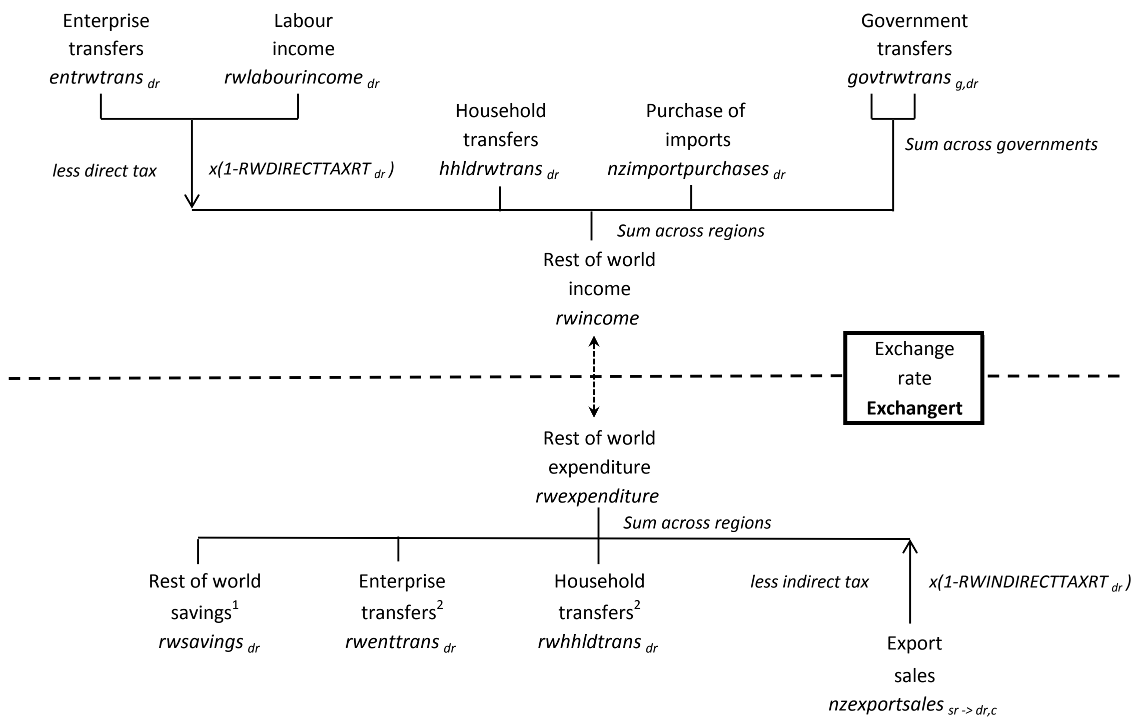

Figure 3.12 Tree diagram showing rest of world income and expenditure.
[1]Exogenously determined, see the Savings & Investment Module (Section 3.9) for details.
[2]The rate at which funds get transferred is exogenously determined and depends on the **Exchangert**.

A tree diagram that provides a summary of the different contributions to rest of world income and expenditure is shown in Figure 3.12. Rest of world income is calculated as the sum of income from labour and transfers from enterprises, less direct taxes, plus income from sale of commodities to NZ (i.e. NZ imports), household transfers, and government transfers (Eq. B.364):

$$rwincome = \sum_{dr} \left(rwlabourincome_{dr} + \sum_{c} \left(nzimportpurchases_{dr,c} \right) + entrwtrans_{dr} \right.$$
$$\left. + hhldrwtrans_{dr} + \sum_{g} \left(govtrwtrans_{g,dr} \right) - rwdirecttax_{dr} \right)$$

Transfers from enterprises, $entrwtrans_{dr}$, households, $hhldrwtrans_{dr}$, and governments, $govtrwtrans_{g,dr}$, are taken directly from the respective modules for these agents, while rest of world labour income is calculated simply by multiplying rest of world labour supply by the relevant labour price (Eqs. B.365, B.366):

$$rwlabourincome_{dr} = \sum_{i} (rwlaboursupply_{dr,i}\, \mathbf{Pfact}_{h=LAB,dr,i})$$

$$rwlaboursupply_{dr,i} = factorsu_{h=LAB,dr,i} \frac{RWFACTRT_{h=LAB,dr}}{1 - RWFACTRT_{h=LAB,dr}}$$

Similarly, rest of world income from imports is determined by multiplying the quantity of import commodities demanded, by the import price (Eq. B.367):

$$nzimportpurchases_{dr,c} = importdemand_{dr,c} \frac{PCOMMWORLDIMP_{c}(t)}{\mathbf{Exchangert}}$$

Finally, rest of world direct taxes are determined by multiplying income from labour and enterprises by an assumed constant direct tax rate, $RWDIRECTTAXRT_{dr}$ (Eq. B.368):

$$rwdirecttax_{dr} = (entrwtrans_{dr} + rwlabourincome_{dr})\, RWDIRECTTAXRT_{dr}$$

On the expenditure side, funds are transferred into the economic system via transfers from enterprises, $entrwtrans_{dr}$, households, $rwhhldtrans_{dr}$, from NZ export sales ,$nzexportsales_{sr,c}$, via payments of indirect taxes, $rwindirecttax_{dr}$, and from net rest of world savings, $rwsavings_{dr}$ (Eq. B.369):

$$rwexpenditure = \sum_{dr} \left(rwsavings_{dr} + rwenttrans_{dr} + rwhhldtrans_{dr} + rwindirecttax_{dr} \right)$$
$$+ \sum_{sr} \sum_{c} (nzexportsales_{sr,c})$$

To calculate indirect taxes, a constant indirect tax rate is applied to NZ's commodity export sales (Eq. B.370):

$$rwindirecttax_{dr} = \sum_{c} (nzexportsales_{sr \to dr,c})\, RWINDIRECTTAXRT_{dr}$$

In turn, export sales of each commodity are calculated as follows (Eqs. B.371, B.99, B.372):

$$nzexportsales_{sr,c} = \frac{expcommoditys_{sr,c}\, actualpexports_{sr,c}}{\mathbf{Exchangert}}$$

$$actualpexports_{sr,c} = \frac{actualexports_{sr,c}\,\mathbf{Pexpcomm}_{sr,c}}{expcommoditys_{sr,c}}$$

$$actualexports_{sr,c} = \min\left(expcommodityd_{sr,c}, expcommoditys_{sr,c}\right)$$

where the quantities of export demands and supply, $expcommodityd_{sr,c}$ and $expcommoditys_{sr,c}$, along with the export commodity price, $\mathbf{Pexpcomm}_{sr,c}$, are available from the commodities module (Section 3.5).

Completing the rest of world module is the calculation of the current account surplus, which is simply total rest of world expenditure less income:

$$actualsurplus = rwexpenditure - rwincome$$

The perceived current account surplus, **Casurplus** will adjust to reflect the actual current account surplus, depending on the time for adjustment, $\tau_{casurplus}$ (Eq. B.224):

$$\frac{d}{dt}\left(\mathbf{Casurplus}\right) = \frac{1}{\tau_{casurplus}}\left(actualcasurplus - \mathbf{Casurplus}\right)$$

3.13 Output variable module

The output variable module contains the calculations for the inflation rate, Consumer Price Index (CPI), and GDP. Although each of these items are also used in calculations elsewhere in the model, the items have been selected for inclusion within this module as these do not fit neatly into the subject matter of the other modules. Furthermore, each item, in its own right, is a useful indicator or reporting item for monitoring the state of an economic system.

If we begin with the inflation rate, this is modelled as a stock that quickly adjusts to match an auxiliary, $targetinflationrt$. In turn, the latter auxiliary is calculated based on the rate of change in the CPI. As with the calculations for rest of world savings, the model also includes an option to override the inflation rate with real data for a historic time period, i.e. as given by the exogenous time series $ACINFLATIONRT(t)$, if appropriate (Eqs. B.373, B.375, B.376):

$$\frac{d}{dt}\left(\mathbf{Inflationrt}\right) = \frac{1}{\tau}\left(targetinflationrt - \mathbf{Inflationrt}\right)$$

$$targetinflationrt = \begin{cases} ACINFLATIONRT(t) & \text{for} \quad t < 11.5 \\ desiredinflationrt & \text{for} \quad t \geq 11.5 \end{cases}$$

$$desiredinflationrt = 4\left(\frac{cpif(t) - cpif(t-0.25)}{cpif(t-0.25)}\right)$$

The CPI is calculated as a Fisher index. The Paasche and Laspeyres indices are alternative approaches for calculating price indices, but both have limitations. On the one hand a Paasche index fails to sufficiently account for substitution within the ‘basket’ of goods considered in the index, while on the other hand the a Laspeyres index tends to over-estimate inflation (IMF, 2004). The Fisher index takes the geometric average of the Paasche and Laspeyres indices, thereby seeking to offset the biases inherent in the two approaches. The CPI Fisher index, $cpif$, is thus calculated as (Eq. B.377):

$$cpif = \sqrt{cpip \times cpil}$$

where $cpip$ and $cpil$ are respectively the CPI Paasche and Laspeyres indices. In turn the CPI Paasche and Laspeyres indices are calculated as follows (Eqs. B.378, B.379):

$$cpil = 1000 \frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, BASEHHLDCONSUMP_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\, BASEHHLDCONSUMP_{dr,c}\right)}$$

$$cpip = 1000 \frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\, hhldconsump_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\, hhldconsump_{dr,c}\right)}$$

In the above equations, $BASEHHLDCONSUMP_{dr,c}$, is, for each region, the quantity of household consumption of each commodity, c, at time zero (the base period), while $BASEPCOMPCOMMD_{dr,c}$, is the associated commodity price at time zero.

In the case of GDP, the Paasche index is used for consistency with national accounts data used in calibration of the model. The quantities and prices that are considered in the calculation of the index are selected based on the expenditure method for calculating GDP (cf Viet, 2009). The method considers a 'basket of goods' that is made up of commodities consumed by households, governments and for investment. The Paasche index is given by (Eq. B.381):

$$\begin{aligned} gdpindexp = &\left[\sum_{dr,c}\left(\left(hhldconsump_{dr,c} + \sum_{g} govtconsump_{g,dr,c} + investconsumpq_{dr,c}\right)\mathbf{Pcompcommd}_{dr,c}\right.\right. \\ &\left.\left. + stockchanges_{dr,c}\right) + \sum_{sr,c} nzexportsales_{sr,c} - \sum_{dr,c} nzimportpurchases_{dr,c}\right] \\ &\div \\ &\left[\sum_{dr,c}\left(\left(hhldconsump_{dr,c} + \sum_{g} govtconsump_{g,dr,c} + investconsumpq_{dr,c}\right.\right.\right. \\ &\left.\left. + \frac{stockchanges_{dr,c}}{\mathbf{Pcompcommd}_{dr,c}}\right) BASECOMPCOMMD_{dr,c}\right) \\ &+ \sum_{sr,c}\left(\frac{nzexportsales_{sr,c}}{pexportcommd_{sr,c}}\mathbf{Exchangert}\right) \\ &\left. - \sum_{dr,c}\left(\frac{nzimportpurchases_{dr,c}}{PCOMMWORLDIMP_{c}(t)}\mathbf{Exchangert} \times 1\right)\right] \end{aligned}$$

where once again, the term '$BASE$' is added to the start of all names of exogenous input data generated from the benchmark or base year accounts (i.e. when time = zero). For example $BASEHHLDCONSUMP_{dr,c}$ is the same as $hhldconsump_{dr,c}$ except that where as the latter is determined for the current time period, the former is generated from the base year accounts.

Note that included in this GDP calculation is the quantity of net commodity exports (i.e. exports less imports). Stock changes, equal to commodities supplied less commodities demanded are also included as an expenditure category in a similar manner to household, government, and investment consumption. Refer to Appendix B.13 for the full set of equations necessary to calculate stock changes.

To calculate the real GDP we must determine the *totalexpenditure*, as it is this value which is divided by the GDP index to obtain GDP in real terms (Eq. B.385):

$$
\begin{aligned}
totalexpenditure = & \sum_{dr,c} \Big[\Big(hhldconsump_{dr,c} + \sum_{g} govtconsump_{g,dr,c} + investconsumpq_{dr,c} \Big) \\
& \times \mathbf{Pcompcommd}_{dr,c} \Big] + \sum_{sr,c} nzexportsales_{sr,c} - \sum_{dr,c} nzimportpurchases_{dr,c} \\
& + \sum_{dr,c} stockchanges_{dr,c} + \sum_{dr} otherindirecttaxes_{dr}
\end{aligned}
$$

The real GDP index that is used in the calculation of the desired interest rate is calculated simply as follows (Eq. B.383):

$$
realgdp = \frac{1000}{actualgdpindex} \times totalexpenditure
$$

GDP is also calculated within the model using the value added or income approach. Unlike the expenditure method, this also allows for reporting at both the industry level within each region, both in nominal ($indvalueadded_{dr,i}$) and real ($dindvalueadded_{dr,i}$) terms (Eqs. B.396, B.397, B.398, B.395, B.394):

$$
indvalueadded_{dr,i} = \sum_{h} (factorsu_{h,dr,i}\, \mathbf{Pfact}_{h,dr,i}) + realindustrybalance_{dr,i} + indindirecttax_{dr,i}
$$

$$
totalvalueadded_{dr} = \sum_{i} indvalueadded_{dr,i} + otherindirecttaxes_{dr}
$$

$$
\begin{aligned}
otherindirecttaxes_{dr} = & \sum_{g} govtindirecttax_{g,dr} + hhldindirecttax_{dr} + rwindirecttax_{dr} \\
& + investindirecttax_{dr}
\end{aligned}
$$

$$
dtotalvalueadded_{dr} = 1000 \frac{totalvalueadded_{dr}}{actualgdpindex}
$$

$$
dindvalueadded_{dr,i} = 1000 \frac{indvalueadded_{dr,i}}{actualgdpindex}
$$

A stock of regional GDP/value added quickly adjusts to match the calculated regional value added in nominal terms ($totalvalueadded_{dr}$). The derivation of a stock avoids computational problems where regional GDP is required to be used elsewhere in the model (Eq. B.374):

$$
\frac{d}{dt} (\mathbf{Holdregvaladd}_{dr}) = \frac{1}{\tau} (totalvalueadded_{dr} - \mathbf{Holdregvaladd}_{dr})
$$

3.14 Freshwater Management Unit and Territorial Local Authority reporting module

This final module in the Southland Economic Model was included to provide capabilities for reporting economic outcomes at sub-regional scales for Southland, namely at the scale of Freshwater Management Unit (FMU) and Territorial Local Authority (TA). Thus far, we have only attempted to report outputs for industry value added and industry employment at these levels.

For the primary industries covered by the primary module, reporting value added and employment results at the level of FMU is relatively straightforward. This is because the primary module keeps track of financial information for each type of farm and mitigation within an economic zone, whilst all economic zones match to one unique FMU. Using information from the primary module (Eqs. B.399 to B.408), calculate for each industry, the share of total value added in the Southland region attributed to each FMU. In the case of non-primary industries the ability to report at sub-regional scales is more difficult. For these industries, it is simply assumed that each FMUs share of an industry's regional value is the same as that estimated for 2017 using sub-regional employment data (see Eq. B.409). Once estimates of value added by industry and FMU are calculated in base year dollar terms, this is inflated to 2017 dollar terms using the GDPSCALAR and also aggregated to a smaller set of industries for reporting (Eqs. B.410 and B.411).

The calculation of industry employment by FMU is similar to the calculation of value added. That is, information from the primary module is used to split primary industry employment at a regional level among primary industries, while for other industries, regional employment is disaggregated among FMUs by using the 2017 FMU shares of regional industry employment (see Eqs. B.402, B.413, B.414, B.415, and B.416).

The generation of reporting indicators for TAs is a little more complicated than for FMUs. This is because while each economic zone is situated within a single FMU, some economic zones are situated across multiple TAs. To deal with this issue, value added/employment for primary industries is first calculated at the level of economic zones. The results for each industry within an economic zone are then allocated across TAs on a pro-rata basis using the relative shares of land in different TAs as calcualted from the 2015 land use map. In other respects the calculation of industry value added and employment at the TA scale works the same as the calculation at the FMU scale. Reference can be made to Eqs. B.417 to B.432, in Appendix B, for the full details.

4 Applying the model

4.1 Input data

Many of the exogenous constants and base (initial condition) data are set from the regional Social Accounting Matrices (SAMs). For the primary module, data is sourced mainly from the FARMAX files created for each individual case study farm and mitigation state as well as from Environment Southland's land use map. Some exogenous constants are calculated in a calibration process. A full list of all exogenous constants and initial conditions is contained in Appendix A, with accompanying descriptions of data sources.

4.2 Scenario settings

The model is a relatively generic economic model that could be used to evaluate a wide range of potential scenarios and policy questions. It may be helpful to think of scenario settings within the model as belonging to two general categories:

- Those settings or parameters that are broadly used to generate the 'reference economic future' under which alternative actions or policies are tested. For example, you may wish to test policies under a reference economic future where there is strong growth in economies in the rest of the world and as a result strong growth in demand for New Zealand commodities.
- The settings or parameters that define the specific policies, investment options and/or behavioural changes that are subject to investigation.

Each of these categories are discussed briefly below.

4.2.1 General scenario settings and reference economic futures

These are a group of input data or settings that broadly determine the economic growth path for NZ. The following sets of input data need to be selected by the model user:

- *Working age population* - the model contains projections of working age population for each region within New Zealand. The default projections are sourced from Statistics New Zealand's medium series, but alternative projections can also be used.
- *Multi-factor productivity* - the model incorporates industry-specific default time series of multi-factor productivity indices $MULTIFACTORPROD_{dr,i}(t)$. These can be altered directly for each individual industry within a region, or a uniform adjustment can be made to all industries' time series through the $ADJUSTRATE_{dr,i}$ parameter.

- *World commodity prices* - the model also incorporates econometric projections of world commodity prices. Export prices, $PCOMMWORLDEXP_c(t)$, are provided separately from import prices, $PCOMMWORLDIMP_c(t)$, on the different sub-commodity compositions of these trades. These default projections can be revised to reflect alternative scenarios.

- *World GDP* - similarly, the default projections for the world GDP index, $WORLDGDPINDX(t)$, are derived via econometric projections but can be altered to reflect new scenarios.

- *Non-productive investments* - the time series $NONPRODINVESTSH(t)$, records the share of investment funds each year that are allocated towards 'non-productive' capital increases. Note that the term 'non-productive' is used in a narrow context here meaning that the additional capital produced does not directly lead to increases in industry output - the investments may well still serve other important objectives such as reducing environmental harm caused by economic production. The default values for this time series are zero.

- *Ad valorem import tariffs* - the time series $ADVALOREMIMPTARIFF_c(t)$, controls additional tariffs that may be applied in a scenario, specified as percentage increases in the current price. Note that since these are only additional tariffs, over and above that already included in the base year SAM, the default values are set to zero.

- *Ad valorem import and export (pseudo) prices* - in the case of exports, we do not model the rest of the world economy and so do not need to model the flow of income generated from tariffs charged in other countries on New Zealand goods. For scenarios where it may be desirable to consider additional export tariffs, the time series $ADVALOREMEXPORTP_c(t)$ (default values set to zero) can be adjusted. This exogenous time series can also be used to implement other situations of changes in international demand caused by factors other than direct price increases, such as changes in quality of goods. In a similar manner, the time series, $ADVALOREMIMPORTP_c(t)$, can be used to alter relative demands for imported goods by the New Zealand market.

- *Labour and capital productivity indices* - the two exogenous time series, $LABINDEX_i(t)$ and $CAPINDEX_i(t)$ (default values set to one), provide further opportunities to adjust the productivity of industries. Each industry's supply of labour is multipled by the LABINDEX, thus providing the 'effective' supply of labour factors, while each industry's stock of capital is multiplied by its CAPINDEX, thereby producing the 'effective' supply of capital for each industry. Note that because of the use of nested CES functions in the model, the changes to the productivity as specified by the multi-factor indices discussed above occur on top of the productivity changes resulting from increases in the LABINDEX and CAPINDEX indices.

- *Additional taxes on production, additional household taxes* - production-based taxes faced by businesses are a key tax mechanism utilised by government. The base SAM already incorporates current levels of production taxes and by default the model assumes that taxes will be paid at the same shares over time. For some scenarios, it may however be necessary to alter taxes, say for example, to capture additional taxes faced by industries based on relative greenhouse gas emissions. The exogenous time series, $INDTAXRTADJUST_i(t)$, provides an opportunity to increase or decrease industry taxes. Additionally, the time series, $HHLDTAXRTADJUST_r(t)$, can be used in scenarios to alter the taxes incurred by households.

4.2.2 Policy settings

As already explained in this report, two policy interfaces have thus far been devised for the model, termed the Municipal Interface and Primary Interface respectively. A full list of the exogenous parameters that are created by each interface is found in Appendix A. When devising these interfaces, attention was given to formulating the levers and controls in such a way that a very wide range of possible policy options could be tested. At the same time, it was necessary to try and keep the interfaces sufficiently simple so that they could be used by people with a wide range of technical/programming skills. Nevertheless, there is always possibility that options will be selected for testing that do not fit nicely into one or other of the interfaces. For example, the Primary Interface may only allow users to input scenarios where farm plans are implemented at a uniform date for all farms within the same farm type but we may wish to test a scenario where different sub-catchments have different implementation dates for farm plans. In such cases it will be necessary to request technical assistance for running the scenarios, likely involving the construction of a bespoke model add-on.

References

Armington, P. 1969. *A Theory of Demand for Products Distinguished by Place of Production.* IMF Staff Papers, 16, 159-178.

Barker, T. 2004. *Economic theory and the transition to sustainability: a comparison of general equilibrium and space-time-economic approaches.* Tyndall Centre for Climatic Change Research. Working Paper 62, Tyndall Centre for Climate Change Research and Applied Economics, University of Cambridge, UK.

Bayoumi, T. with assistance from Laxton, D., Faruqee, H., Hunt, B., Karam, P., Lee, J., Rebucci, A. and Tchakarov, I. 2004. *GEM: a New International Macroeconomic Model.* International Monetary Fund, Washington, DC.

Brown, C., Giovianazzi, S., Seville, E., Vargo, J., Stevenson, J.R. (2015) *Developing the Business Behaviours Module within MERIT.* ERI Research Report 2015/001, GNS Science, Wellington.

Creedy, J., Gemmell, N., Scobie, G. 2015. Pensions, Savings and Housing: A Life-cycle Framework with Policy Simulations. Working papers in Public Finance 01/2015. Victoria University, Wellington.

Forrester, J. 1961. *Industrial Dynamics.* MIT Press and John Wiley and Sons, Inc. New York.

Forrester, J. 1969. *Urban Dynamics.* MIT Press, Cambridge.

Forrester, J. 1971. *World Dynamics.* Wright-Allen Press, Massachusetts.

Grassini, M. 2004. *Rowing along the general equilibrium modelling mainstream.* Paper presented Ecomode conference on "Input-Output and General Equilibrium: Data, Modelling and Policy Analysis", Brussels. September 2-4.

Hertel, T.W., Narayanan, B. and McDougall, R.A. 2012. Behavioural Parameters, Chapter 14 in Narayanan, G., Agular, A. and McDougall, R. (eds.) *Global Trade, Assistance, and Production: The GTAP 8 Data Base* Center for Global Trade Analysis, Purdue University.

Horridge, J.M., Zhai, F. 2005. "Shocking a Single-Country CGE Model with Export Prices and Quantities from a Global Model" annex to Chapter 3 in T.W. Hertel and Winter, L.A. (eds) *Poverty and the WTO: Impacts of the Doha Development Agenda* Palgrave Macmillan, pp.183-217.

International Monetary Fund (IMF), 2004. *Producers Price Index and Manual: Theory and Practice.* IMF, Washington, DC.

Johansen, L. 1960. *A Multi-Sector Study of Economic Growth*, North-Holland, Amsterdam.

Kalra, N., Hallegatte, S., Lempert, R., Brown, C., Fozzard, A., Stuart, G., Shah, A. 2014. *Agreeing on robust decisions: new processes for decision making under deep uncertainty.* Policy Research Working Paper 6906. The World Bank, Climate Change Group, Office of the Chief Economist.

Rae, A. and Strutt, A. 2005. *Agricultural trade reform and nitrogen pollution in OECD countries.* Agricultural Policy Discussion Paper No 19. Massey University, Palmerston North.

Rae, A. and Strutt, A. 2011. *Modelling the Impacts of Policies to Reduce Environmental Impacts in the New Zealand Dairy Sector.* Working Paper in Economics 04/11, Department of Economics, University of Waikato, Hamilton.

Richardson, G.P. 2011. Reflections on the foundations of system dynamics. System Dynamics Review, 27, 219-243.

Robson, N. 2012 *Multi-Regional Computable General Equilibrium Model for New Zealand: A thesis submitted to the Victoria University of Wellington in fulfilment of the requirements for the degree of Doctor of Philosophy.* Victoria University, Wellington.

Scrieciu, S. S. 2007. *The inherent dangers of using computable general equilibrium models as a single integrated modelling framework for sustainability impact assessment.* A critical note on Bohringer and Loschel (2006). Ecological Economics, 60, 678-684.

Smith, N.J., McDonald, G.W. 2007. *Ecological Footprints of New Zealand and its Regions 2003-04.* Report prepared for Ministry for the Environment. Market Economics, Takapuna, Auckland.

Smith, N.J., Zhang, Y., Cardwell, R., McDonald, G., Kim, J-H, Murray, C. 2015. Development of a Social Accounting Framework for New Zealand. ERI Research Report 2015/01, GNS Science, Lower Hutt.

Taylor, J. 1993. *Discretion Versus Policy Rules in Practice.* Carnegie-Rochester Conference Series on Public Policy. Vol.39 (December), 195-214.

Viet, V.Q. 2009. *GDP by production approach: A general introduction with emphasis on an integrated economic data collection framework.* Available online at: http://unstats.un.org/unsd/.

A Index of names and definitions

A.1 Subscripts and concordances

Most subscripts are described in Table 2.1 on page 7. However, the natural capital types, and industry and commodity groupings can vary between different model applications and are described here.

Table A.1 Natural capital types

Name	Description
$NatCap1$	Agricultural/forestry land
$NatCap2$	Coal
$NatCap3$	Oil/gas

Table A.2 Industry categories

Name	Description
$Ind1$	Sheep, beef, deer, other livestock and grain farming horticulture
$Ind2$	Dairy cattle farming
$Ind3$	Forestry and logging
$Ind4$	Other primary
$Ind5$	Meat and meat product manufacturing
$Ind6$	Dairy product manufacturing
$Ind7$	Other food manufacturing
$Ind8$	Textile and leather manufacturing
$Ind9$	Wood and paper manufacturing
$Ind10$	Other manufacturing
$Ind11$	Utilities
$Ind12$	Construction
$Ind13$	Wholesale and retail trade
$Ind14$	Accommodation and food services
$Ind15$	Transport, postal, courier, transport support and warehousing services
$Ind16$	Finance, insurance, rental, real estate services and business services

(continued on next page)

Table A.2 (continued)

Name	Description
$Ind17$	Ownership of owner-occupied dwellings
$Ind18$	Government, education, health and social services
$Ind19$	Personal and recreational services

Table A.3 Commodity categories

Name	Description
$Com1$	Trade margins
$Com2$	Horticulture and fruit
$Com3$	Forage products, fibres, sugar crops and unmanufactured tobacco
$Com4$	Cereals
$Com5$	Sheep
$Com6$	Cattle
$Com7$	Deer
$Com8$	Other livestock and animal products
$Com9$	Raw milk
$Com10$	Wool
$Com11$	Wood and non-wood forest products and standing timber
$Com12$	Coal, oil, gas and other mining and quarrying
$Com13$	Meat products
$Com14$	Dairy products
$Com15$	Other food
$Com16$	Animal feed
$Com17$	Textiles
$Com18$	Wood and paper products
$Com19$	Other manufactures
$Com20$	Utilities
$Com21$	Construction
$Com22$	Accommodation and restaurant services
$Com23$	Transport services, storage and warehousing
$Com24$	Finance, insurance, rental and leased property services, real estate services and business services
$Com25$	Owner-occupied dwellings
$Com26$	Government, education, health and social services
$Com27$	Personal and recreational services

Table A.4 Agricultural industry categories

Name	Description
$AgIn01$	Sheep and beef
$AgIn02$	Deer
$AgIn03$	Dairy cattle farming
$AgIn04$	Forestry and logging
$AgIn05$	Arable
$AgIn06$	Horticulture

Table A.5 IO Agricultural industry categories

Name	Description
$IOAg01$	Sheep, beef, deer, other livestock and grain farming horticulture
$IOAg02$	Dairy cattle farming
$IOAg03$	Forestry and logging

Table A.6 Reporting industry categories

Name	Description
$ShBfDr$	Sheep, beef, deer, other livestock and grain farming horticulture
$DaCaFm$	Dairy cattle farming
$ForLog$	Forestry and logging
$OtPrim$	Other primary
$DaManuf$	Dairy product manufacturing
$OtFoManuf$	Other food manufacturing
$OtManuf$	Other manufacturing
$UtCoTrt$	Utilities, construction and transport
$TrHosp$	Trade and hospitality
$FiInReBu$	Finance, insurance, real estate and business services
$OtServ$	Other services

Table A.7 Farm type categories

Name	Description
$FaTyp01$	Sheep & Beef A
$FaTyp02$	Sheep & Beef B

(continued on next page)

Table A.7 (continued)

Name	Description
$FaTyp03$	Sheep & Beef C
$FaTyp04$	Sheep & Beef D
$FaTyp05$	Sheep & Beef E
$FaTyp06$	Sheep & Beef F
$FaTyp07$	Sheep & Beef G
$FaTyp08$	Sheep & Beef H
$FaTyp09$	Deer A
$FaTyp10$	Deer B
$FaTyp11$	Deer C
$FaTyp12$	Deer D
$FaTyp13$	Deer E
$FaTyp14$	Dairy cattle farming
$FaTyp15$	Forestry and logging
$FaTyp16$	Horticulture
$FaTyp17$	Arable

Table A.8 Mitigation state categories

Name	Description
$Miti01$	Baseline
$Miti02$	Dairy farming: 10% N reduction
	Sheep, beef and deer farming: Nutrient inputs
	Arable farming: N mitigation
	Horticulture: 10% N fertiliser reduction
$Miti03$	Dairy farming: 20% N reduction
	Sheep, beef and deer farming: Crop policy
	Arable: Stock removal
	Horticulture: 20% N fertiliser reduction
$Miti04$	Dairy farming: 30% N reduction
	Sheep, beef and deer farming: Stock policy
	Horticulture: 30% N fertiliser reduction
$Miti05$	Dairy farming: 40% N reduction
	Sheep, beef and deer farming: Fence pacing and wallowing
$Miti06$	Dairy farming: 10% P reduction
$Miti07$	Dairy farming: 20% P reduction

(continued on next page)

Table A.8 (continued)

Name	Description
$Miti08$	Dairy farming: 30% P reduction
$Miti09$	Dairy farming and Sheep, beef and deer farming: Land retirement to forestry

Table A.9 Case study farm categories

Name	Description
$MoFa01$	Sheep, beef and dairy case study farm 1
$MoFa02$	Sheep, beef and dairy case study farm 2
$MoFa03$	Sheep, beef and dairy case study farm 3
$MoFa04$	Sheep, beef and dairy case study farm 4
$MoFa05$	Sheep, beef and dairy case study farm 5
$MoFa06$	Sheep, beef and dairy case study farm 7
$MoFa07$	Sheep, beef and dairy case study farm 8
$MoFa08$	Sheep, beef and dairy case study farm 9
$MoFa09$	Sheep, beef and dairy case study farm 10
$MoFa10$	Sheep, beef and dairy case study farm 11
$MoFa11$	Sheep, beef and dairy case study farm 12
$MoFa12$	Sheep, beef and dairy case study farm 13
$MoFa13$	Sheep, beef and dairy case study farm 14
$MoFa14$	Sheep, beef and dairy case study farm 15
$MoFa15$	Sheep, beef and dairy case study farm 16
$MoFa16$	Sheep, beef and dairy case study farm 17
$MoFa17$	Sheep, beef and dairy case study farm 18
$MoFa18$	Sheep, beef and dairy case study farm 19
$MoFa19$	Sheep, beef and dairy case study farm 20
$MoFa20$	Sheep, beef and dairy case study farm 21
$MoFa21$	Sheep, beef and dairy case study farm 22
$MoFa22$	Sheep, beef and dairy case study farm 23
$MoFa23$	Sheep, beef and dairy case study farm 24
$MoFa24$	Sheep, beef and dairy case study farm 25
$MoFa25$	Sheep, beef and dairy case study farm 27
$MoFa26$	Sheep, beef and dairy case study farm 28
$MoFa27$	Sheep, beef and dairy case study farm 29
$MoFa28$	Sheep, beef and dairy case study farm 30
$MoFa29$	Sheep, beef and dairy case study farm 31

(continued on next page)

Table A.9 (continued)

Name	Description
$MoFa30$	Sheep, beef and dairy case study farm 32
$MoFa31$	Sheep, beef and dairy case study farm 34
$MoFa32$	Sheep, beef and dairy case study farm 35
$MoFa33$	Sheep, beef and dairy case study farm 36
$MoFa34$	Sheep, beef and dairy case study farm 37
$MoFa35$	Sheep, beef and dairy case study farm 38
$MoFa36$	Sheep, beef and dairy case study farm 39
$MoFa37$	Sheep, beef and dairy case study farm 40
$MoFa38$	Sheep, beef and dairy case study farm 41
$MoFa39$	Sheep, beef and dairy case study farm 42
$MoFa40$	Sheep, beef and dairy case study farm 43
$MoFa41$	Sheep, beef and dairy case study farm 44
$MoFa42$	Sheep, beef and dairy case study farm 45
$MoFa43$	Sheep, beef and dairy case study farm 46
$MoFa44$	Sheep, beef and dairy case study farm 47
$MoFa45$	Dairy economic zone 1
$MoFa46$	Dairy economic zone 2
$MoFa47$	Dairy economic zone 3
$MoFa48$	Dairy economic zone 4
$MoFa49$	Dairy economic zone 5
$MoFa50$	Dairy economic zone 6
$MoFa51$	Dairy economic zone 7
$MoFa52$	Dairy economic zone 8
$MoFa53$	Dairy economic zone 9
$MoFa54$	Dairy economic zone 10
$MoFa55$	Horticulture
$MoFa56$	Arable
$MoFa57$	Forestry

Table A.10 Freshwater Management Unit categories

Name	Description
$Apar$	Aparima
$Mata$	Mataura
$Oret$	Oreti

(continued on next page)

Table A.10 (continued)

Name	Description
$TeAn$	Waiau - Te Anau
$WaSo$	Waiau - South
$FiLd$	Fiordland and Islands

Table A.11 Territorial Authority categories

Name	Description
$South$	Southland District
$Gore$	Gore District
$Inver$	Invercargill City

Table A.12 Economic zone categories

Name	Description
$Zone01$	Aparima large farm
$Zone02$	Aparima, small, mixed, well drained, wet farm
$Zone03$	Aparima, small, mixed, poorly drained, wet farm
$Zone04$	Aparima, small, flat, well drained, wet farm
$Zone05$	Aparima, small, flat, poorly drained, wet farm
$Zone06$	Aparima, small, flat, well drained, dry farm
$Zone07$	Aparima, small, flat, poorly drained, dry farm
$Zone08$	Mataura large farm
$Zone09$	Mataura, small, mixed, well drained, wet farm
$Zone10$	Mataura, small, mixed, poorly drained, wet farm
$Zone11$	Mataura, small, mixed, well drained, dry farm
$Zone12$	Mataura, small, mixed, poorly drained, dry farm
$Zone13$	Mataura, small, flat, well drained, wet farm
$Zone14$	Mataura, small, flat, poorly drained, wet farm
$Zone15$	Mataura, small, flat, well drained, dry farm
$Zone16$	Mataura, small, flat, poorly drained, dry farm
$Zone17$	Oreti large farm
$Zone18$	Oreti, small, mixed, well drained, wet farm
$Zone19$	Oreti, small, mixed, poorly drained, wet farm
$Zone20$	Oreti, small, flat, well drained, wet farm

(continued on next page)

Table A.12 (continued)

Name	Description
$Zone21$	Oreti, small, flat, poorly drained, wet farm
$Zone22$	Oreti, small, flat, well drained, dry farm
$Zone23$	Oreti, small, flat, poorly drained, dry farm
$Zone24$	Waiau - Te Anau large farm
$Zone25$	Waiau - Te Anau, small, mixed, well drained, wet farm
$Zone26$	Waiau - Te Anau, small, mixed, poorly drained, wet farm
$Zone27$	Waiau - Te Anau, small, flat, well drained, wet farm
$Zone28$	Waiau - Te Anau, small, flat, poorly drained, wet farm
$Zone29$	Waiau - South large farm
$Zone30$	Waiau - South, small, mixed, well drained, wet farm
$Zone31$	Waiau - South, small, mixed, poorly drained, wet farm
$Zone32$	Waiau - South, small, flat, well drained, wet farm
$Zone33$	Waiau - South, small, flat, poorly drained, wet farm

Table A.13 Municipal funding categories

Name	Description
$Hhld$	Household
$Buss$	Business
$Cgovt$	Central government

Table A.14 Rate types

Name	Description
$Normal$	Normal
$FinIntServ$	Financial Intermediate Services

Table A.15 Year mitigated

Name
$Year01$
$Year02$
$Year03$
$Year04$

(continued on next page)

Table A.15 (continued)

Name
$Year05$
$Year06$
$Year07$
$Year08$
$Year09$
$Year10$
$Year11$
$Year12$
$Year13$
$Year14$
$Year15$
$Year16$
$Year17$
$Year18$
$Year19$
$Year20$
$Year21$
$Year22$
$Year23$
$Year24$
$Year25$

A.2 Stocks

Table A.16 Description of stocks and equation references

Name	Description	Eq. ref
$\mathbf{Builtcapital}_{dr,i}$	Built capital	(B.178)
Casurplus	Current account surplus	(B.224)
$\mathbf{Desiredprod}_{dr,i}$	Value of desired industry production	(B.55)
$\mathbf{Estimports}_{dr,c}$	Commodity imports	(B.78)
Exchangert	Exchange rate	(B.362)
$\mathbf{Holdregvaladd}_{dr}$	Total regional value added	(B.374)
$\mathbf{Indlaboursup}_{dr,i}$	Supply of labour to industries	(B.167)

(continued on next page)

Table A.16 (continued)

Name	Description	Eq. ref
$\textbf{Industryaccount}_{dr,i}$	Available industry funds	(B.56)
$\textbf{Industrybalance}_{dr,i}$	Difference between industry income and industry expenditure	(B.57)
Inflationrt	Inflation rate	(B.373)
Interestrt	Interest rate	(B.225)
$\textbf{Landuse}_{ez,ft,mt}$	Land use	(B.263)
$\textbf{Loantopay}_{yrmt,rt,mf,lt}$	Loan to pay	(B.246)
$\textbf{Naturalcapital}_{dr,nct}$	Natural capital	(B.179)
$\textbf{Pbuiltcap}_{dr,i}$	Industry built capital price	(B.180)
$\textbf{Pcfact}_{dr,i}$	Industry composite factor price	(B.146)
$\textbf{Pcindustrys}_{sr,i}$	Composite industry supply price	(B.79)
$\textbf{Pcompcommd}_{dr,c}$	Composite commodity demand price	(B.80)
$\textbf{Pcompcomms}_{sr,c}$	Composite commodity supply price	(B.81)
$\textbf{Pcompdomcommd}_{dr,c}$	Composite domestic commodity demand price	(B.82)
$\textbf{Pcompdomcomms}_{sr,c}$	Composite domestic commodity supply price	(B.83)
$\textbf{Pcompnaturalcapd}_{dr,i}$	Composite natural capital demand price	(B.182)
$\textbf{Pcompnaturalcaps}_{dr,nct}$	Composite natural capital supply price	(B.183)
$\textbf{Percfsreturnsperha}_{ez,ft,mt}$	Perceived returns per hectare	(B.262)
$\textbf{Pexpcomm}_{sr,c}$	Price of export commodities	(B.84)
$\textbf{Pfact}_{h,dr,i}$	Price of factors	(B.147)
$\textbf{Pfiinputs}_{dr,i}$	Price of composite inputs	(B.85)
$\textbf{Pgovtcc}_{g,dr}$	Perceived government composite commodity consumption price	(B.27)
$\textbf{Phhldcc}_{dr}$	Household composite commodity consumption price	(B.2)
$\textbf{Pintinputs}_{dr,i}$	Price of composite intermediate inputs	(B.86)
$\textbf{Pinvestcc}_{dr}$	Investment composite commodity consumption price	(B.226)
$\textbf{Pnaturalcap}_{dr,i,nct}$	Price of natural capital types	(B.181)
$\textbf{Pperceivedcompcommd}_{dr,c}$	Perceived composite domestic supply price	(B.88)
$\textbf{Pregdomcomm}_{sr,dr,c}$	Price of commodities produced in and for the domestic market	(B.87)
$\textbf{Rcapincome}_{dr}$	Recognised capital income	(B.184)
$\textbf{Renterincome}_{dr}$	Recognised enterprise income	(B.45)
$\textbf{Rgovtincome}_{g,dr}$	Recognised government income	(B.26)
$\textbf{Rhhldincome}_{dr}$	Recognised household income	(B.1)
$\textbf{Shareretirefr}_{ez,agi}$	Share of land retired to forestry	(B.261)
$\textbf{Shareretirenp}_{ez,agi}$	Share of land retired to non-productive use	(B.260)

Table A.17 Initial condition settings for stocks

Name	Notes
$iBuiltcapital_{dr,i}$	Net capital stock by 31 industry types obtained from Statistics New Zealand (series SG07NAC04K90). This is disaggregated to 106 industries on a pro rata basis according to each industry's share of total capital factor payments. The national industry estimates are then further disaggregated to 15 NZ regions on a pro rata basis according to the regional share of each industry's total consumption of fixed capital. The latter data is obtained from multi-regional supply use tables (Smith *et al.*, 2015)
$iCasurplus$	Set to 0 in base year
$iDesiredprod_{dr,i}$	Set to BASEPRODUCTION in base year
$iEstimports_{dr,c}$	Set to BASEIMPORTS in base year
$iExchangert$	Set to 1 in base year
$iGdpindex$	Set to 1000 in base year
$iIndlaboursup_{dr,i}$	Employment by industry and (demand) region obtained from Statistics New Zealand's Annual Business Frame (http://www.stats.govt.nz/). This data is disaggregated to regions from which employment sourced (supply region) according to the proportion of total labour factor payments from each region allocated to each supply region, as specified in the multi-regional supply and use tables (Smith *et al.*, 2015)
$iIndustryaccount_{dr,i}$	Derived from base year Social Accounting Matrix (SAM)
$iIndustrybalance_{dr,i}$	Set to 0 in base year
$iInflationrt$	Set as $ACINFLATIONRT(0)$
$iInterestrt$	Set as $ACTUALINTERESTRT(0)$
$iLanduse_{ez,ft,mt}$	Landuse by economic zone and broad primary use cateogry (sheep/beef, deer, dairy, arable, horticulture, forestry) derived from Environment Southland's land use map. For sheep/beef and deer, total land use by economic zone is split between Farm Types based on estimated proportions given by industry experts
$iNaturalcapital_{dr,nct}$	Total agricultural land data ($NatCap1$) at September 2006 is estimated from StatisticsNZ Agricultural Census (hectares) long term timeseries. Coal and Oil/gas resources ($NatCap2$ and $NatCap3$, respectively) are derived from Smith and McDonald (2007)
$iPbuiltcap_{dr,i}$	Prices in base year set to 1
$iPcindustrys_{sr,i}$	Prices in base year set to 1
$iPcfact_{dr,i}$	Prices in base year set to 1
$iPcompcommd_{dr,c}$	Prices in base year set to 1
$iPcompcomms_{sr,c}$	Prices in base year set to 1
$iPcompdomcommd_{dr,c}$	Prices in base year set to 1
$iPcompdomcomms_{sr,c}$	Prices in base year set to 1
$iPcompnaturalcapd_{dr,i}$	Prices in base year set to 1
$iPcompnaturalcaps_{sr,i}$	Prices in base year set to 1
$iPercfsreturnsperha_{ez,ft,mt}$	Derived from farm financial accounts for case study farms, with adjustments for price changes between the year financial accounts compiled (2015) and base year of model
$iPexpcomm_{sr,c}$	Prices in base year set to 1

(continued on next page)

Table A.17 (continued)

Name	Notes
$iPfact_{h,dr,i}$	Prices in base year set to 1
$iPfiinputs_{dr,i}$	Prices in base year set to 1
$iPgovtcc_{g,dr}$	Prices in base year set to 1
$iPhhldcc_{dr}$	Prices in base year set to 1
$iPintinputs_{dr,i}$	Prices in base year set to 1
$iPinvestcc_{dr}$	Prices in base year set to 1
$iPnaturalcap_{dr,i,nct}$	Prices in base year set to 1
$iPregdomcomm_{sr,dr,c}$	Prices in base year set to 1
$iRcapincome_{dr}$	Derived from base year SAM
$iRegvaladd_{dr}$	Initial total regional value added, derived from base year SAM
$iRenterincome_{dr}$	Derived from base year SAM
$iRgovtincome_{g,dr}$	Derived from base year SAM
$iRhhldincome_{dr}$	Set to BASEHHLDACCOUNT in base year

A.3 Auxiliaries

Table A.18 Description of auxiliaries and equation references

Name	Description	Eq. ref
$actualagriprod_{dr,i}$	Actual primary industry production	(B.164)
$actualcasurplus$	Actual current account surplus	(B.227)
$actualenterincome_{dr}$	Current enterprise income	(B.46)
$actualexports_{sr,c}$	Current commodity exports	(B.372)
$actualgdpindex$	Current Fisher Gross Domestic Product (GDP) index	(B.380)
$actualhhldincome_{dr}$	Current household income	(B.3)
$actualindcompnaturalcaps_{dr,i}$	Current supply of composite natural capital to industries	(B.216)
$actuallanduse_{ez,ft,mt}$	Actual adjustment to land use to account for known changes in land use	(B.264)
$actualpcapital_{dr,i}$	Current capital price	(B.151)
$actualpccd_{dr,c}$	Current composite commodity demand price	(B.109)
$actualpccs_{sr,c}$	Current composite commodity supply price	(B.112)
$actualpcdcd_{dr,c}$	Current composite domestic commodity demand price	(B.114)
$actualpcdcs_{sr,c}$	Current composite domestic industry supply price	(B.116)
$actualpcfact_{dr,i}$	Current composite factor price	(B.148)

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Table A.18 (continued)

Name	Description	Eq. ref
$actualpcnaturalcapd_{dr,i}$	Current composite natural capital demand price	(B.215)
$actualpcnaturalcaps_{dr,nct}$	Current composite natural capital supply price	(B.212)
$actualpercccd_{dr,c}$	Actual perceived price of composite commodity demand	(B.136)
$actualpexports_{sr,c}$	Current exports price	(B.99)
$actualpfiinputs_{dr,i}$	Current composite factor and intermediate inputs price	(B.120)
$actualpgovtcc_{g,dr}$	Current government composite commodity consumption price	(B.38)
$actualphhldcc_{dr}$	Current household composite commodity consumption price	(B.17)
$actualpintinputs_{dr,i}$	Current composite intermediate inputs price	(B.124)
$actualpinvestcc_{dr}$	Current investment composite commodity consumption price	(B.231)
$actualprod_{dr,i}$	Actual production	(B.60)
$actualsupply_{sr,i,c}$	Actual supply of commodities by industry	(B.69)
$addtravelcosts_{dr}$	Additional household travel costs	(B.433)
$adjustedindlaboursup_{dr,i}$	Industry labour supply adjusted for additional productivity	(B.177)
$adoptionrate_{ez,ft,mt}$	Adoption rate for sigmoidal technological adoption curve	(B.331)
$adoptionsteepness_{ez,ft,mt}$	Steepness of technological adoption curve	(B.330)
$adoptiontype_{ez,ft,mt}$	Type of adoption (0=no adoption, 1=early adoption, 2=medium adoption, 3=late adoption)	(B.324)
$aggregateinvestv1_{dr}$	Total value of investment, first estimate	(B.235)
$aggregateinvestv2_{dr}$	Total value of investment, second estimate with adjustments for committed investment in municipal wastewater treatment	(B.236)
$agindshareofallocation_{ez,agi}$	Share of land subject to land use change allocated to each primary industry as a new land use (final estimate)	(B.297)
$agindshareofallocation1st_{ez,agi}$	Share of land subject to land use change allocated to each primary industry as a new land use (first estimate)	(B.296)
$agrilandsup_{dr,i,nct}$	Agricultural/forestry land supply by industry	(B.223)
$agriprodrt_{IOag}$	Rate of production from primary industries	(B.351)
$annualloanpayments_{rt,lt}$	Annual load payments	(B.252)
$avgreturnsperha_{ez,agi}$	Average returns per hectare	(B.294)
$belowcap_{ez,ft,mt}$	Status of land use in relation to land use cap (1=below land use cap, 0=at or above land use cap)	(B.320)
$betwgovttransin_{g,dr}$	Within-region transfers between government agents, by paying agent	(B.33)
$betwgovttransout_{g,dr}$	Within-region transfers between government agents, by receiving agent	(B.34)

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Table A.18 (continued)

Name	Description	Eq. ref
$bopratio$	Balance of payments ratio	(B.363)
$builtcaprevsh_{ft}$	Share of revenue allocated to built capital	(B.270)
$builtratio_{dr,i}$	Ratio of industry built capital supply to industry built capital demand	(B.185)
$builts_{dr,i}$	Supply quantity of built capital	(B.186)
$builtuse_{dr,i}$	Built capital used	(B.205)
$bussloanpayments_{dr}$	Loan payments by industries	(B.219)
$capentertrans_{dr}$	Transfers of capital income to enterprises	(B.192)
$capgovttrans_{g,dr}$	Transfers of capital income to government	(B.193)
$capitalincome_{dr}$	Total capital income	(B.190)
$capincomehhld_{dr}$	Transfers of capital income to households, by receiving region	(B.4)
$capincomesh_{dr,i}$	Industry share of total regional capital income	(B.203)
$capitaltyped_{cap,dr,i}$	Industry capital demand	(B.208)
$caplocalhhldtrans_{dr}$	Capital income transferred to within-region households	(B.194)
$capreghhldtrans_{dr}$	Capital income transferred to out-of-region households, by region of capital income receipt	(B.195)
$capregtransout_{dr}$	Capital income transferred between regions, by receiving region	(B.191)
$ccapitals_{dr,i}$	Composite capital supply	(B.187)
$centralgovmunpays_{g,dr}$	Additional municipal costs (loan payments + operating costs) allocated to central government	(B.44)
$compfactord_{dr,i}$	Industry demand for composite factors	(B.71)
$compfactoru_{dr,i}$	Industry use of composite factors	(B.157)
$compnaturalcaps_{dr,i}$	Composite natural capital supply	(B.189)
$councilfeeprice$	Council fee price	(B.282)
$cpif$	Fisher Consumer Price Index (CPI)	(B.377)
$cpil$	Laspeyres CPI	(B.378)
$cpip$	Paasche CPI	(B.379)
$deadlinepressure_{ez,ft,mt}$	Deadline pressure	(B.332)
$depreciation_{dr,i}$	Depreciation	(B.206)
$desiredinflationrt$	Desired inflation rate	(B.376)
$desiredinterestrt$	Desired interest rate	(B.228)
$dfmuindva_{fmu,i}$	Deflated (to base year dollar terms) value added by industry and Freshwater Management Unit	(B.409)

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Table A.18 (continued)

Name	Description	Eq. ref
$dindvalueadded_{dr,i}$	Industry value added deflated to constant (base year) dollar terms	(B.394)
$directtaxincome_{g,dr}$	Government income from direct taxes	(B.29)
$disinvestconsump_{dr,c}$	Investment consumption by commodity	(B.234)
$domcomdemand_{dr,c}$	Domestic commodity demand	(B.111)
$domcommexpend_{dr,i,c}$	Value of industry expenditure on domestic commodities	(B.65)
$domcommoditys_{sr,c}$	Domestic commodity supply	(B.91)
$domcommodityuse_{dr,i,c}$	Domestic commodity use of commodities	(B.67)
$dommarginq_{dr}$	Quantity of domestic margins	(B.437)
$dtaindva_{ta,i}$	Deflated (to base year year dollar terms) value added by industry and Territorial Authority	(B.426)
$dtotalvalueadded_{dr}$	Total regional domestic product calculated based on value-added method and deflated to constant (base year) dollar terms	(B.395)
$effectcompfactord_{dr,i}$	Effective composite factor demand during a disruption that affects operability	(B.72)
$effectcompfactoru_{dr,i}$	Effective composite factor use during a disruption that affects operability	(B.161)
$effectfactorsd_{h,dr,i}$	Effective factor demand during a disruption that affects operability	(B.159)
$effectfactorss_{h,dr,i}$	Effective factor supply during a disruption with reduced operability	(B.160)
$effectfactorsu_{h,dr,i}$	Effective factor use during a disruption that affects operability	(B.158)
$entdirecttax_{dr}$	Enterprise direct taxes	(B.47)
$entgovttrans_{g,dr}$	Enterprise transfers to government	(B.51)
$enthhldtrans_{dr}$	Enterprise transfers to households	(B.52)
$entincomehhld_{dr}$	Total enterprise transfers to households, by receiving region	(B.5)
$entreghhldtrans_{dr}$	Enterprise income transferred to out-of-region households, by region of enterprise income receipt	(B.53)
$entregtransout_{dr}$	Transfers between enterprises within different regions, by paying region	(B.50)
$entrwtrans_{dr}$	Transfers from enterprises to the rest of the world	(B.49)
$entsavtrans_{dr}$	Enterprise savings	(B.54)
$excessproduction_{sr,dr,c}$	Excess of commodity production over use	(B.100)
$expagindreturnsperha_{ez,agi}$	Expected primary industry returns per hectare	(B.293)
$expcommodityd_{sr,c}$	Export commodity demand	(B.97)

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Table A.18 (continued)

Name	Description	Eq. ref
$expcommoditys_{sr,c}$	Export commodity supply	(B.90)
$exportmargindemand_{sr,m}$	Demand for transport margins on exports	(B.438)
$exportmarginsupply_{dr,m}$	Supply of transport margins on exports	(B.439)
$exportratio_{sr,c}$	Ratio of export supply to demand	(B.89)
$factinputshare_{dr,i}$	Share of total inputs comprised of factors	(B.122)
$factinputunitcost_{dr,i}$	Unit cost of factor inputs	(B.75)
$factorsd_{h,dr,i}$	Demand for factors	(B.155)
$factorss_{h,dr,i}$	Supply of factors	(B.154)
$factorsu_{h,dr,i}$	Use of factors	(B.156)
$factscalep1_{dr,i}$	Scale parameter for the CES function for composite factors, with primary industries adjusted for any changes calculated in the primary module	(B.163)
$factsharep1_{h,dr,i}$	Share parameter for the CES function for composite factors, with primary industries adjusted for changes calculated in the primary module	(B.162)
$farmcontragindreturns_{ez,ft,agi}$	Expected returns by farm type, weighted by current proportion of land in relevant agricultural industry and zone currently allocated to farm type	(B.292)
$farmcontrzonereturns_{ez,ft}$	Expected returns by farm type, weighted by current proportion of land in relevant zone currently allocated to farm type	(B.303)
$farmindustryland_{ft}$	Total land area in region allocated to each farm type	(B.304)
$farmretirement_{ez,ft,mt}$	Land retired from primary production	(B.310)
$fenceprice$	Fencing service price	(B.281)
$fencingcostsbytimeai_{fmu,agi}$	Fencing costs by time and by agricultural industry where fencing takes place	(B.312)
$fencingcostsbytimeioag_{fmu,IOag}$	Fencing costs by time and by IO agricultural industry where fencing takes place	(B.313)
$ffmitinputcoefha_{ez,ft,c}$	Farm forestry mitigation input coefficient per hectare	(B.284)
$ffmitoutputcoefha_{ez,ft,c}$	Farm forestry mitigation output coefficient per hectare	(B.286)
$ficapitalinput_{cap,IOag}$	Total quantity of capital inputs for primary industries	(B.274)
$fifactorinput_{h,IOag}$	Total quantity of each factor inputs for primary industries	(B.273)
$fifiinputcoef_{input,IOag}$	Input coefficient for factor and intermediate inputs for primary industries	(B.275)
$fiintinputcoef_{IOag,c}$	Input coefficient for intermediate inputs for primary industries	(B.277)
$fioutput_{IOag,c}$	Total commodity outputs by primary industries	(B.288)

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Table A.18 (continued)

Name	Description	Eq. ref
$finintservdemands_{mf,lt}$	Quantity of financial intermediate services demanded through loans funding new municipal wastewater treatment	(B.247)
$fisalecominput_{IOag}$	Revised scale parameter for primary industries in the CES function for composite intermediate inputs	(B.340)
$fiscalecapitalinput_{IOag}$	Revised scale parameter for primary industries in the CES function for composite capital inputs	(B.343)
$fiscalecomsup_{IOag}$	Revised scale parameter for primary industries in the CET function for composite commodity supply	(B.348)
$fiscalefactorinput_{IOag}$	Revised scale parameter for primary industries in the CES function for composite factor inputs	(B.342)
$fiscalefiinput_{IOag}$	Revised scale parameter for primary industries in the CES function for composite factor and intermediate inputs	(B.341)
$fisharecapitalinput_{IOag,cap}$	Revised share parameter for primary industries in the CES function for composite capital inputs	(B.339)
$fisharecominput_{IOag,c}$	Revised share parameter for primary industries in the CES function for composite intermediate inputs	(B.335)
$fisharecomsup_{IOag,c}$	Revised share parameter for primary industries in the CET function for composite commodity supply	(B.336)
$fisharefactorinput_{IOag,h}$	Revised share parameter for primary industries in the CES function for composite factor inputs	(B.338)
$fisharefiinput_{IOag,input}$	Revised share parameter for primary industries in the CES function for composite intermediate and factor inputs	(B.337)
$fmuagindva_{fmu,IOag}$	First estimate of primary industry value added by Freshwater Management Unit	(B.406)
$fmufarmcapital_{fmu,ft}$	Total returns to capital by Freshwater Management Unit and farm type	(B.404)
$fmufarmlab_{fmu,ft}$	Total labour factor payments by Freshwater Management Unit and farm type	(B.401)
$fmufarmlabIOind_{fmu,IOag}$	Total labour factor payments by Freshwater Management Unit and primary industry	(B.402)
$fmuindemployment_{fmu,i}$	Employment by industry and Freshwater Management Unit	(B.415)
$fmuindustryva2017_{fmu,i}$	Value added by industry and Freshwater Management Unit in 2017 dollar terms	(B.410)
$fmureportindemp_{fmu,ri}$	Employment by reporting industry and Freshwater Management Unit	(B.416)
$fmureportindva_{fmu,ri}$	Value added by reporting industry and Freshwater Management Unit	(B.411)
$fmushareagindva_{fmu,IOag}$	Freshwater Management Unit share of regional value added for primary industries	(B.407)
$fmushareindempagri_{fmu,ft}$	Freshwater Management Unit share of regional employment for primary industries	(B.413)

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Table A.18 (continued)

Name	Description	Eq. ref
$fmushareindvaagri_{fmu,i}$	Freshwater Management Unit share of regional value added for all industries	(B.408)
$fmushemploy_{fmu,i}$	Freshwater Management Unit share of employment for all industries	(B.414)
$forestryintensityscalar_{ez,ft}$	Forestry land use intensity	(B.280)
$fsreturnsperha_{ez,ft,mt}$	Returns per hectare	(B.289)
$fspermittedreturns_{ez,ft,mt}$	Perceived returns per hectare (set to very low value for farm type mitigations that are not permitted to increase in land area)	(B.321)
$fsshareallocation_{ez,ft,mt,agi}$	Farm type and mitigation allocation share of new land uses	(B.299)
$gdpindexp$	Paasche GDP index	(B.381)
$gdpgap$	Difference between perceived and actual GDP index	(B.229)
$gdppercapita$	GDP per capita	(B.384)
$govtcompcommd_{g,dr}$	Government composite commodity demand	(B.42)
$govtconsump_{g,dr,c}$	Government commodity consumption	(B.40)
$govtdirecttax_{g,dr}$	Direct taxes on government	(B.31)
$govthhldtrans_{g,dr}$	Transfers from government to households	(B.35)
$govtincome_{g,dr}$	Government income	(B.28)
$govtindirecttax_{g,dr}$	Indirect taxes on government	(B.43)
$govtrwtrans_{g,dr}$	Transfers from government to the rest of the world	(B.32)
$govtsavings_{g,dr}$	Government savings	(B.36)
$grossreturn_{dr,i}$	Gross returns on capital	(B.202)
$hhldcompcommd_{dr}$	Household composite commodity demand	(B.21)
$hhldconsump_{dr,c}$	Household commodity consumption	(B.19)
$hhldconsumprt_{dr}$	Household consumption rate	(B.16)
$hhlddirecttax_{dr}$	Direct taxes on households	(B.23)
$hhldenttrans_{dr}$	Transfers from households to enterprises	(B.10)
$hhldgovttrans_{g,dr}$	Transfers from households to government	(B.11)
$hhldindirecttax_{dr}$	Indirect taxes on households	(B.22)
$hhldindtaxrtadjusted_{dr}$	Household indirect tax rate with adjustments	(B.24)
$hhldloanpayments_{dr}$	Household loan payments for new investments in municipal wastewater treatment	(B.25)
$hhldregtransout_{dr}$	Transfers between households within different regions, by paying region	(B.12)
$hhldrwtrans_{dr}$	Transfers from households to the rest of the world	(B.8)
$hhldsavings_{dr}$	Household savings	(B.14)

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Table A.18 (continued)

Name	Description	Eq. ref
$hhldtotal_{dr}$	Remaining household income after transfers and additional fixed costs (e.g. travel margins)	(B.13)
$immobileinvest_{dr,i}$	Investment that is fixed to industry responsible for capital income	(B.198)
$importcommexpend_{dr,i,c}$	Expenditure by industries on commodities	(B.64)
$importdemand_{dr,c}$	Demand for commodity imports	(B.103)
$importmargindemand_{dr,m}$	Demand for margins on imports	(B.440)
$importmarginsupply_{sr,m}$	Supply of margins on imports	(B.441)
$importtariffs_{g,dr}$	Value of tax income received by government from import tariffs	(B.135)
$indcapincome_{dr,i}$	Current rate of income on capital, by industry	(B.204)
$indcommoditys_{sr,i,c}$	Supply of commodities by industries	(B.94)
$indconsump_{dr,i,c}$	Use of commodities by industries	(B.106)
$indexpendu_{dr,i}$	Industry expenditure	(B.63)
$indfencingcosts_{fmu,i}$	Industry fencing costs	(B.346)
$indindirecttax_{dr,i}$	Indirect taxes on industries	(B.76)
$indindirecttaxrtadjusted_{dr,i}$	Adjusted indirect tax rate on industries	(B.77)
$indirecttaxincome_{g,dr}$	Government income from direct taxes on industries	(B.30)
$indnaturalcaps_{dr,i,nct}$	Industry supply of natural capital, final estimate	(B.211)
$indplancosts_{fmu,i}$	Industry plan costs	(B.347)
$indnaturalcaps1_{dr,i,nct}$	Supply of natural capital to industries, first estimate	(B.213)
$induseshare_{dr,i,c}$	Each industry's share of total commodity use by industries	(B.66)
$industryinc_{dr,i}$	Industry income	(B.68)
$industryland_{IOag}$	Total agriculture/forestry land use by primary industries	(B.349)
$indvalueadded_{dr,i}$	Industry value added	(B.396)
$initialpyear_{agi}$	Defines whether time is within the initial year for farm plans (1= yes, 0=no)	(B.318)
$interinputshare_{dr,i}$	Share of industry inputs comprised of intermediate inputs	(B.123)
$interinputunitcost_{dr,i}$	Unit cost of intermediate inputs	(B.74)
$intinputcoeff_{dr,i,c}$	Commodity share of intermediate inputs	(B.126)
$investadjustcap$	Adjustment to regional investment fund relating to physical capital purchases for new municipal wastewater treatment	(B.254)
$investadjustland$	Adjustment to regional investment fund relating to land purchases for new municipal wastewater treatment	(B.256)
$investconsumpq_{dr,c}$	Quantity of commodities consumed for investment	(B.243)
$investindirecttax_{dr}$	Indirect taxes on investment	(B.244)

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Table A.18 (continued)

Name	Description	Eq. ref
$ioindfencingplancosts_{fmu,IOag}$	Total fencing and farm plan costs	(B.423)
$labincomedregion_{dr}$	Total labour income by region of income generation	(B.7)
$labincomesupply_{sr}$	Total labour income paid to households, by region of labour supply	(B.6)
$labourforce_{sr}$	Labour force	(B.175)
$labratio_{dr,i}$	Ratio of labour supply to labour demand	(B.153)
$labtoreallocate_{dr,i}$	Labour potentially reallocated between industries	(B.169)
$landintensityscalar_{ez,agi}$	Land intensity scalar	(B.278)
$landintensityscalar2_{ez,ft}$	Land intensity scalar	(B.279)
$landnomitigation_{ez,ft}$	Land with no available mitigation for current farm type	(B.300)
$landshare_{ez,ft,mt,agi}$	Share of zone's current land use	(B.305)
$landtoreallocate_{ez}$	Land to reallocate	(B.301)
$landusechangein_{ez,ft,mt}$	Land added under land use change	(B.307)
$landusechangeout_{ez,ft,mt}$	Land subtracted under land use change	(B.326)
$loanpayments_{yrmt,rt,mf,lt}$	Loan payments	(B.249)
$loanpaymentsbyyearloan_{yrmt,rt,mf,lt}$	Annual load payments by year loan commenced	(B.251)
$managsystlabcoefha_{ez,ft,mt}$	Labour coefficient per hectare by farm type and mitigation	(B.267)
$managsystoutputcoefha_{ez,ft,mt,c}$	Output coefficient per hectare by farm type and mitigation	(B.287)
$mangsystdepreccoefha_{ez,ft,mt}$	Depreciation coefficient per hectare by farm type and mitigation	(B.268)
$mangsystindtaxcoefha_{ez,ftmt}$	Industry tax coefficient per hectare by farm type and mitigation	(B.266)
$mangsystinputcoefha_{ez,ft,mt,c}$	Commodity input coefficient per hectare by farm type and mitigation	(B.285)
$marginconsumpq_{dr,c}$	Consumption of margin commodities	(B.442)
$maxloanpayment_{yrmt,rt,mf,lt}$	Maximum annual loan payments	(B.259)
$maxpermittedreturns_{ez,ft}$	Maximum returns per hectare among those land uses within a zone that are permitted to increase in size	(B.317)
$maxprod_{dr,i}$	Maximum production under operability constraints	(B.434)
$maxprodsup_{dr,i}$	Maximum production	(B.61)
$mecforce_{sr}$	Available labour measured in modified employment counts (MECs)	(B.174)
$mfpadjusted_{dr,i}$	Adjusted multifactor productivity	(B.166)
$midpoint_{ez,ft,mt}$	Midpoint of the technological adoption curve	(B.328)
$mitigationin_{ez,ft,mt}$	New land added to a mitigation state	(B.309)
$mitigationout_{ez,ft,mt}$	Land lost from a mitigation state	(B.311)

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Table A.18 (continued)

Name	Description	Eq. ref
$mobileinvest_{dr,i}$	Mobile investment	(B.197)
$mobileinvest1_{dr,i}$	First estimate of mobile investment	(B.200)
$mobileinvestsh_{dr,i}$	Industry share of total mobile investment	(B.199)
$modfarmitffinputcoefha_{mofa,c}$	Selected commodity input coefficient for farm forestry mitigation (land not under forestry)	(B.360)
$modfarmmitffoutputcoefha_{mofa,c}$	Selected commodity output coefficient for farm forestry mitigaton (land not under forestry)	(B.361)
$modfarmoscoefha_{ez,ft,mt}$	Operating surplus coefficient per ha	(B.269)
$multifactorprod2_{dr,i}$	Multifactor productivity adjusted for changes in productivity calculated with primary module	(B.165)
$muninvestconsump_{dr,c}$	Consumption of investment commodities for new municipal wastewater upgrades	(B.255)
$naturalcapitalsq_{dr,i}$	Supply quantity of natural capital in normalised units	(B.188)
$naturalcapd_{dr,i,nct}$	Demand for natural capital	(B.207)
$naturalcapratio_{dr,i,nct}$	Ratio of natural capital supply to demand	(B.210)
$netcapitalchange_{dr,i}$	Net capital change associated with primary industry land use change	(B.222)
$netincreaselab_{dr}$	Net increase in labour supply	(B.171)
$netratelandusechange_{IOag}$	Net land use change per total land use	(B.350)
$netreturn_{dr,i}$	Net return on investment	(B.201)
$newcapital_{dr,i}$	New capital	(B.196)
$newlabsupply_{dr,i}$	New labour supplied to industries	(B.170)
$netlandusechange_{ez,ft}$	Net land use change	(B.306)
$noticeperiod_{ez,ft,mt}$	Period of notice for rule requiring mitigation	(B.327)
$noticeperiodscalar$	Notice period scalar	(B.329)
$numberpayments$	Number of loan payments	(B.257)
$nzexportsales_{sr,c}$	Value of commodity exports	(B.371)
$nzimportpurchases_{dr,c}$	Value of commodity imports	(B.367)
$otherindirecttaxes_{dr}$	Other indirect taxes from government, household, investment, and rest of world	(B.398)
$perceivedpexportd_{sr,c}$	Perceived price of export commodities	(B.137)
$perceivedimportp_{dr,c}$	Perceived price of import commodities	(B.138)
$periodicloanpayments_{rt,lt}$	Loan payment each loan pay period	(B.253)
$periodicrate_{rt,lt}$	Periodic interest rate	(B.258)
$permittedland_{ez,ft}$	Land area in farm types that have at least one mitigation that is a permitted option for land use change	(B.298)

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Table A.18 (continued)

Name	Description	Eq. ref
$permittednow_{ez,ft}$	Defines if land in a particular farm type and mitigation is permitted recipient of land use change (0=no, 1= yes)	(B.323)
$pexpcommnz_{sr,c}$	Price of export commodities in NZ currency	(B.92)
$pexportcommd_{sr,c}$	Price of export commodities including margins	(B.98)
$pexportmargins_{sr,c}$	Price of export margins	(B.448)
$pexprailmargins_{sr,dr}$	Price of rail margins on composite commodity demand	(B.446)
$pexproadmargins_{sr,dr}$	Price of road margins on composite commodity demand	(B.447)
$pfarmbuiltcapital_{ft}$	Price of built capital for primary industries	(B.358)
$pfarmlabour_{ft}$	Price of labour for primary industries	(B.359)
$pimpcommnz_{dr,c}$	Price of import commodities in NZ currency	(B.108)
$pimportmargins_{dr,c}$	Price of import margins	(B.445)
$pimporttariffs_{c}$	Price of tariffs on import commodities	(B.134)
$pimprailmargins_{sr,dr}$	Price of rail margins on commodities produced in and for the domestic market	(B.443)
$pimproadmargins_{sr,dr}$	Price of road margins on commodities produced in and for the domestic market	(B.444)
$plancost1bytime_{fmu,agi}$	Farm plan costs, consenting related	(B.314)
$plancost2bytime_{fmu,agi}$	Farm plan costs, other than consenting related	(B.315)
$plancostsbytimeioag_{fmu,IOag}$	Total farm plan costs faced by primary industries	(B.316)
$potentialsales_{dr,i}$	Potential value of industry sales	(B.70)
$pregdomcomminclmargin_{sr,dr,c}$	Price of regional domestic commodities including margins	(B.449)
$prodadjust_{ft}$	Adjusted productivity by farm type	(B.357)
$qcapitald_{dr,i}$	Quantity of capital demand	(B.152)
$qcompcommd_{dr,c}$	Quantity of composite commodity demand	(B.110)
$qcompcomms_{sr,c}$	Quantity of composite commodity supply	(B.113)
$qcompfactd_{dr,i}$	Quantity of composite factor demand	(B.149)
$qasplannedprod_{dr,i}$	Available quantity of production - based on production at onset of disruption	(B.435)
$qdesiredprod_{dr,i}$	Quantity of production desired	(B.436)
$qdomcommd_{dr,c}$	Quantity of domestic commodity demand	(B.115)
$qdomcomms_{sr,c}$	Quantity of domestic commodity supply	(B.117)
$qfiinputs_{dr,i}$	Quantity of composite factor and intermediate input supply	(B.121)
$qgovtcc_{g,dr}$	Quantity of composite government consumption	(B.39)
$qhhldcc_{dr}$	Quantity of composite household consumption	(B.18)
$qintinputs_{dr,i}$	Quantity of composite intermediate inputs	(B.125)

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Table A.18 (continued)

Name	Description	Eq. ref
$qinvestcc_{dr}$	Quantity of composite investment consumption	(B.232)
$realgdp$	Real GDP	(B.383)
$realindustrybalance_{dr,i}$	Difference between industry income and industry expenditure	(B.62)
$realinterestrt$	Real interest rate	(B.230)
$reallocatedlab_{dr,i}$	Labour reallocated from other industries	(B.168)
$regcdomcomms_{sr,dr,c}$	Domestic commodity supply, by region of commodity production (sr) and commodity consumption (dr)	(B.101)
$regcommoditys_{sr,c}$	Total commodity production within NZ regions	(B.93)
$regdomcommd_{sr,dr,c}$	Domestic commodity demand, by region of commodity production (sr) and commodity consumption (dr)	(B.102)
$regindprodincltax_{sr,i}$	Composite industry production	(B.96)
$reglabourest_{sr,dr}$	Labour supply, by region of labour origin (sr) and labour use (dr), first estimate	(B.173)
$reglaboursupply_{sr,dr}$	Labour supply, by region of labour origin (sr) and labour use (dr)	(B.172)
$regsavings_{dr}$	Total regional savings	(B.241)
$residualcap_{IOag}$	Residual capital	(B.403)
$residuallab_{IOag}$	Residual labour	(B.405)
$rulenotified_{ez,ft,mt}$	Defines whether a rule restricting a mitigation state is notified	(B.322)
$rwdirecttax_{dr}$	Rest of world direct tax	(B.368)
$rwenttrans_{dr}$	Transfers from the rest of the world to enterprises	(B.48)
$rwexpenditure$	Total expenditure for the rest of the world	(B.369)
$rwhhldtrans_{dr}$	Transfers from the rest of the world to households	(B.9)
$rwincome$	Total income for the rest of the world	(B.364)
$rwindirecttax_{dr}$	Total indirect taxes on exports	(B.370)
$rwlabourincome_{dr}$	Rest of world labour income	(B.365)
$rwlaboursupply_{dr,i}$	Rest of world labour supply	(B.366)
$rwsavings_{dr}$	Rest of world savings, by NZ region	(B.239)
$rwsavingstotal$	Total rest of world savings	(B.240)
$savingstotal1_{dr}$	First estimate of total regional savings	(B.237)
$savingstotal2_{dr}$	Second estimate of total regional savings, adjusted for loan payments required for new investments in municipal wastewater treatment	(B.238)
$savregtransout_{dr}$	Transfers of savings between regions, by paying region	(B.242)

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Table A.18 (continued)

Name	Description	Eq. ref
$scalecc1_{dr,i}$	Scale parameter for the CES function for composite capital, adjusted for changes to primary industries as calculated in the primary module	(B.221)
$scalecominput1_{dr,i}$	Scale parameter for the CES function for commodity inputs, adjusted for changes to primary industries as calculated in the primary module	(B.140)
$scalecommsup1_{sr,i}$	Scale parameter for the CET function for commodity outputs, adjusted for changes to primary industries as calculated in the primary module	(B.142)
$scaledhhldincome_{dr}$	Household income scaled to 2017 dollar terms	(B.412)
$scalefi1_{dr,i}$	Scale parameter for the CES function for composite factor and intermediate inputs, adjusted for changes to primary industries as calculated in the primary module	(B.144)
$selectedfs_{ez,ft,mt}$	Defines whether a mitigation state is selected as the recipient of land use change if land within same farm type is required to adopt a different mitigation	(B.325)
$sharecapitaldisag_{IOag,cap}$	Intermediate step in the calculation of revised share parameters for capital inputs for primary industries	(B.334)
$sharecc1_{cap,dr,i}$	Share parameter for the CES function for composite capital, adjusted for changes to primary industries as calculated in the primary module	(B.220)
$sharecominput1_{dr,i,c}$	Share parameter for the CES function for composite commodity inputs, adjusted for changes to primary industries as calculated in the primary module	(B.139)
$sharecommsup1_{sr,i,c}$	Share parameter for the CET function for composite commodity outputs, adjusted for changes to primary industries as calculated in the primary module	(B.143)
$sharefiinputdisag_{IOag,input}$	Intermediate step in the calculation of revised share parameters for factors and intermediate inputs for primary industries	(B.333)
$sharefi1_{input,dr,i}$	Share parameter for the CES function for composite intermediate inputs and factors, adjusted for changes to primary industries as calculated in the primary module	(B.141)
$shareofadoptions_{ez,ft,mt}$	Share of land in farm type required to adopt a new mitigation allocated to mitigation state	(B.291)
$stockchanges_{dr,c}$	Value of commodity production less commodity use	(B.382)
$subpyear_{agi}$	Defines whether time is post the year farm plans are first adopted (1=yes,0=no)	(B.319)
$supcoeff_{sr,i,c}$	Commodity share of composite industry production	(B.95)
$taagindcap_{ta,agi}$	Total returns to capital for primary industries, by Territorial Local Authority	(B.419)

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Table A.18 (continued)

Name	Description	Eq. ref
$taagindlab_{ta,agi}$	Total payments to labour for primary industries, by Territorial Local Authority	(B.420)
$taindemployment_{ta,i}$	Employment by industry and by Territorial Local Authority	(B.431)
$taindustryva2017_{ta,i}$	Value added by industry and by Territorial Local Authority, scaled to 2017 dollar terms	(B.427)
$taioagindcap_{ta,IOag}$	Returns to capital for primary industries, by Territorial Local Authority	(B.421)
$taioagindlab_{ta,IOag}$	Payments to labour for primary industries, by Territorial Local Authority	(B.422)
$taioagindva_{ta,IOag}$	Value added by primary industries by Territorial Local Authority	(B.424)
$tareportindemp_{ta,i}$	Employment by reporting industry, by Territorial Local Authority	(B.432)
$tareportindval_{ta,ri}$	Value added by reporting industry, by Territorial Local Authority	(B.428)
$targetinflationrt$	Target inflation rate	(B.375)
$tashareioagriemp_{ta,IOag}$	Share of industry employment attributed to Territorial Local Authority, primary industries only	(B.429)
$tashareindvaagri_{ta,i}$	Share of primary industry value added attributed to Territorial Local Authority, mapped to full model industries	(B.425)
$tashemploy_{ta,i}$	Share of industry employment attributed to Territorial Local Authority	(B.430)
$taylorinterestrt$	Modelled interest rate	(B.245)
$totalcomdemand_{dr,c}$	Total regional commodity demand	(B.105)
$totalcommodityexpend_{dr,c}$	Total industry expenditure on commodities both from domestic and imported origin	(B.393)
$totalexpenditure$	Total domestic expenditure	(B.385)
$totalfencingdemand_{dr,c}$	Total demand for commodities for fencing	(B.345)
$totalfibuiltcapital_{IOag}$	Total use of built capital factor inputs by primary industries	(B.271)
$totalfiintinput_{ez,ft,mt}$	Total use of composite factors and composite intermediate inputs by primary industries	(B.276)
$totalfilabour_{IOag}$	Total use of labour factor inputs by primary industries	(B.265)
$totalfiland_{IOag}$	Total use of land factor inputs by primary industries	(B.272)
$totalfslandmitigated_{ez,ft}$	For a farm type and zone, quantity of land adopting a new mitigation state	(B.308)
$totalgovtconsump_{g,dr}$	Total value of government consumption	(B.37)
$totalhhldconsump_{dr}$	Total value of household consumption	(B.15)
$totalindconsump_{dr,c}$	Total value of industry consumption	(B.107)

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Table A.18 (continued)

Name	Description	Eq. ref
$totallanduse_{ez,agi}$	Total land use	(B.302)
$totalloanpayments_{rt,mf,lt}$	Total loan payments	(B.248)
$totalplandemand_{dr,c}$	Total demand for commodities for farm plan creation	(B.344)
$totaltopay_{yrmt,rt,mf,lt}$	Total loans to pay	(B.250)
$totalvalueadded_{dr}$	Total value added	(B.397)
$unavailablelab_{sr}$	Unavailable labour	(B.176)
$unitcost_{dr,i}$	Unit cost of production	(B.73)
$vcomdemand_{sr,c}$	Value of commodity demand	(B.59)
$vcommoditydemand_{dr,c}$	Total value of commodity demand	(B.389)
$vcommoditysupply_{dr,c}$	Total value of commodity supply	(B.386)
$vgovtconsump_{dr,c}$	Value of government consumption	(B.391)
$vhhldconsump_{dr,c}$	Value of household consumption	(B.390)
$viableffmit_{ez,ft,mt}$	Defines whether farm forestry mitigation is viable	(B.290)
$vinddemandc_{sr,i,c}$	Value of industry demand for each commodity	(B.58)
$vinvestconsump_{dr,c}$	Value of investment consumption	(B.392)
$vmunopex_{dr,c,mf}$	Value of operating expenditure for new municipal wastewater treatment, by entity responsible for funding	(B.218)
$zoneagindcap_{ez,agi}$	Returns to capital for primary industries, by zone	(B.417)
$zoneagindlab_{ez,agi}$	Payments for labour for primary industries, by zone	(B.418)
$zonefarmcapital_{ez,ft}$	Returns to capital by farm types, by zone	(B.400)
$zonefarmlab_{ez,ft}$	Payments for labour by farm types, by zone	(B.399)
$\eta^{cc}_{dr,i}$	Parameter for input substitution between capital types	(B.209)
$\eta^{com}_{dr,c}$	Parameter for input substitution between domestic and imported commodities	(B.127)
$\eta^{cominput}_{dr,i}$	Parameter for input substitution between different types of commodities	(B.128)
$\eta^{fact}_{dr,i}$	Parameter for input substitution between factors	(B.150)
$\eta^{fi}_{dr,i}$	Parameter for input substitution between composite factors and composite intermediate inputs	(B.129)
$\eta^{ficapital}_{IOag}$	Parameter for input substitution between built and land capitals for primary industries	(B.352)
$\eta^{ficominput}_{IOag}$	Parameter for input substitution between commodities for primary industries	(B.353)
$\eta^{fifactinput}_{IOag}$	Parameter for input substitution between labour and capital factors for primary industries	(B.354)

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Table A.18 (continued)

Name	Description	Eq. ref
$\eta^{fifiinput}_{IOag}$	Parameter for input substitution between intermediate inputs and factors for primary industries	(B.355)
$\eta^{govtc}_{g,dr}$	Parameter for commodity substitution in government consumption	(B.41)
η^{hhldc}_{dr}	Parameter for commodity substitution in household consumption	(B.20)
$\eta^{investc}_{dr}$	Parameter for commodity substitution investment consumption	(B.233)
$\eta^{natcap}_{dr,i}$	Parameter for input substitution between natural capital types	(B.217)
$\eta^{regcom}_{dr,c}$	Parameter for input substitution between commodities produced in different NZ regions	(B.130)
$\phi^{com}_{sr,c}$	Parameter for transformation of commodity supply between export and domestic markets	(B.132)
$\phi^{comoutput}_{IOag}$	Parameter for transformation of industry supply between different types of commodities, for primary industries	(B.356)
$\phi^{comsup}_{sr,c}$	Parameter for transformation of industry supply between different types of commodities	(B.131)
ϕ^{fland}_{ez}	Parameter for transformation of total land available for land use change into each type of land use	(B.295)
$\phi^{natcap}_{dr,nct}$	Parameter for transformation of natural capital supply between different industries	(B.214)
$\phi^{regcom}_{sr,c}$	Parameter for transformation of commodity supply between different regional commodity markets	(B.133)

A.4 Exogenous inputs

Table A.19 Description of exogeneous constants

Name	Description
$ADJUSTRATE$	*Annual rate of adjustment to multi-factor productivity projections* Set to -0.0005 for base case (derived as part of model calibration), but can be adjusted for different scenarios
$ADJUSTTIME$	*Time to adjust perceptions of primary system returns per hectare* Set to 2.5 years
$ALLOCATESH_{dr}$	*Share of total mobile investment that is allocated to industries based on the relative returns to capital in those industries.*

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Table A.19 (continued)

Name	Description
	Set to one less the estimated proportion of investment in the base year that is assigned to industries with negative net returns
$ALPHA$	*Weight given to the real interest weight in determining the total value of investment.* Estimated from linear regression of historic data
$BASECOMMFLOW_{sr,dr,c}$	*Commodities used by (dr) regions and sourced from production (sr) regions during the base year.* Derived from base year SAM
$BASECONSUMPRT_{dr}$	*Rate of household consumption during the base year.* Derived from base year SAM
$BASEEXPORTS_{dr,c}$	*Value of commodity exports during the base year.* Derived from base year SAM
$BASEGDP_{dr}$	*GDP for the base year.* Derived from base year SAM
$BASEGOVTCONSUMP_{g,dr,c}$	*Government commodity consumption for the base year.* Derived from base year SAM
$BASEHHLDACCOUNT_{dr}$	*Household income available for current consumption and savings during the base year.* Derived from base year SAM
$BASEHHLDCONSUMP_{dr,c}$	*Household commodity consumption for the base year.* Derived from base year SAM
$BASEINVESTCONSUMP_{dr,c}$	*Investment commodity consumption for the base year.* Derived from base year SAM
$BASEPCOMPCOMMD_{dr,c}$	*Base year composite commodity demand price.* Derived from base year SAM
$BASEPRODUCTION_{dr,i}$	*Base year industry production.* Derived from base year SAM
$BASEREALINTERESTRT$	*Base year real interest rate.* Set at 4.2% (www.rbnz.govt.nz/statistics/)
$BETA$	*Weight given to the value of total regional savings in determining the total value of investment.* Estimated from linear regression of historic data
$BTWGOVTTRANSRT_{g,dr}$	*Rate of transfers between government agents.* Derived from base year SAM
$CENTTRANSRT_{dr}$	*Share of capital income transferred to enterprises.* Derived from base year SAM

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Table A.19 (continued)

Name	Description
$CGOVTTRANSRT_{g,dr}$	*Share of capital income transferred to government.* Derived from base year SAM
$CHHLDTRANSRT_{dr}$	*Share of household income transferred to households.* Derived from base year SAM
$CIRELASTICITY_{dr}$	*Parameter that controls the degree to which household consumption changes in response to changes in the real interest rate.* Set at -0.075 (Creedy *et al.*, 2015)
$CONVERSIONRT_{dr,nct}$	*Rate of change of total natural capital stocks* Set at -0.72% for $NatCap1$ (agricultural land) based on Agricultural Production Statistics timeseries trends.
$CREGTRANSRT_{dr}$	*Share of capital income transferred out of the region.* Derived from base year SAM
$CRHTRANSRT_{dr}$	*Share of capital income transferred to households out of the region.* Derived from base year SAM
$DEPSHFT$	*Adjustment to depreciation rate.* Default parameters derived from model calibration
$DIRECTTAXSH_{g,dr}$	*Share of total regional direct tax income allocated to government agents.* Derived from base year SAM
$EGOVTTRANS$	*Parameter that controls the degree to which transfers from government to the rest of the world change in response to changes in the current account surplus.* Set to 1
$EGOVTTRANSRT_{g,dr}$	*Government savings rate.* Derived from base year SAM
$EHHLDTRANSRT_{dr}$	*Share of enterprise income transferred to households.* Derived from base year SAM
$EINVEST_{dr,i}$	*Parameter that controls the degree to which investment allocated to industries responds to changes in the rate of return on capital.* Derived from model calibration
$ENTTAXRT_{dr}$	*Enterprise direct tax rate.* Derived from base year SAM
$EREGTRANSRT_{dr}$	*Share of enterprise income transferred out of the region.* Derived from base year SAM
$ERHHLDTRANSRT_{dr}$	*Share of enterprise income transferred to households located out of the region.*

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Table A.19 (continued)

Name	Description
	Derived from base year SAM
$ERWTRANSBS_{dr}$	*Transfers from enterprises to the rest of the world during the base year.* Derived from base year SAM
$ESAVTRANSRT_{dr}$	*Share of enterprise income transferred to savings.* Derived from base year SAM
$EXPORTP_{c}$	*Parameter that controls the change in export demand in response to the price of NZ export commodities relative to the world price.* Derived from Horridge and Zhai (2005)
$FARMPERHA_{fmu,agi}$	*Average number of farms per hectare* Data sourced from Environment Southland
$FCSHENTRWTRANS$	*Foreign currency share of transfers from enterprises to the rest of the world.* Set at 0.5
$FCSHHHLDRWTRANS_{dr}$	*Foreign currency share of transfers from households to the rest of the world.* Set at 0.5
$FCSHRWENTTRANS_{dr}$	*Foreign currency share of transfers from the rest of the world to enterprises.* Set at 0.5
$FCSHRWHHLDTRANS_{dr}$	*Foreign currency share of transfers from the rest of the world to households.* Set at 0.5
$FMUSHAREEMP_{fmu,i}$	*Share of regional industry employment within each Freshwater Management Unit.* Calculated from data on Modified Employment Counts by Meshblock for 2017, Sourced from Market Economics BD Deluxe Database
$FMUSHRESIDUAL_{fmu,IOag}$	*Share of residual employment in any industry allocated to each Freshwater Management Unit.* Estimated from the ratio of land use in a Freshwater Management Unit excluded from any economic zone in the model to the total land use in all Freshwater Management Units excluded from any economic zone in the model
$FMUTAMAP_{ta,fmu,IOag}$	*For each primary industry, share of total effective land use in a Freshwater Management Unit located within each Territorial Authority boundary.* Calculated from land use data provided by Environment Southland for 2015

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Table A.19 (continued)

Name	Description
$FMUZONEMAP_{fmu,ez}$	*Maps relationship between economic zones and Freshwater Management Units: Value of 0 assigned if economic zone is outside Freshwater Management Unit, value of 1 assigned if economic zone is within Freshwater Management Unit.* Derived from zone definitions
$FUNEMPLOYRT$	*Frictional unemployment rate.* Set at 3.5% as this is the lowest level of unemployment achieved in recent times and likely reflects 'full employment'
$GDPSCALAR$	*GDP Price Inflator.* Sourced from SNZ series: Implicit price deflator, seasonally adjusted, Total (Qrtly-Mar/Jun/Sep/Dec)
$GDPWEIGHT$	*Weight given to the world GDP index in determining the value of savings from the rest of the world.* Estimated from linear regression of historic data
$GDPPARAM_c$	*Parameter that determines extent to which demand for exports increases with increases in world GDP.* Derived from model calibration
$GOVTCONSUMPRT_{g,dr}$	*Share of government income allocated to consumption.* Derived from base year SAM
$GOVTDIRECTTAXRT_{g,dr}$	*Government direct tax rate.* Derived from base year SAM
$GOVTHHLDTRANSRT_{g,dr}$	*Share of government income transferred to households.* Derived from base year SAM
$GOVTINDIRECTTAXRT_{g,dr}$	*Indirect tax rate on government commodity consumption.* Derived from base year SAM
$GOVTRWTRANSBS_{g,dr}$	*Transfers from government to the rest of the world during the base year.* Derived from base year SAM
$GOVTSAVRT_{g,dr}$	*Rate of savings on government income.* Derived from base year SAM
$HHLDENTTRANSRT_{dr}$	*Share of household income transferred to enterprises.* Derived from base year SAM
$HHLDGOVTTRANSRT_{g,dr}$	*share of household income transferred to government.* Derived from base year SAM
$HHLDINDTAXRT_{dr}$	*Indirect tax rate on household commodity consumption.* Derived from base year SAM

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Table A.19 (continued)

Name	Description
$HHLDREGTRANSRT_{dr}$	*Share of household income transferred to households located outside of the local region.* Derived from base year SAM
$HHLDRWTRANSBS_{dr}$	*Transfers from households to the rest of the world during the base year.* Derived from base year SAM
$HHLDSAVINGADJUST_{dr}$	*Adjustment to household budget to reflect household spending above household income.* Derived from base year SAM
$HOLDLANDTIME$	*Time post which land uses are held constant.* Normally set to a time post the end of a model run but can be set to an alternative time if it is desirable to investigate model runs when land use is held constant
$INDINDIRECTTAXRT_{dr,i}$	*Indirect tax rate on industry commodity consumption.* Derived from base year SAM
$INDIRECTTAXSH_{g,dr}$	*Share of indirect tax income allocated to government agents.* Derived from base year SAM
$INTERESTCONST$	*Interest rate constant in calculation of desired interest rate (post GFC).* Estimated from linear regression of historic data
$INTERESTCONSTGFC$	*Interest rate constant in calculation of desired interest rate (pre GFC).* Estimated from linear regression of historic data
$INTERESTGDPW$	*Parameter applied to GDP gap in the calculation of desired interest rate.* Estimated from linear regression of historic data
$INTERESTINFLW$	*Parameter applied to inflation rate gap in the calculation of desired interest rate.* Estimated from linear regression of historic data
$INVESTCONST$	*Constant term in investment function.* Derived from model calibration
$INVESTCONSTSH_{dr,i}$	*Industry share of regional investment held constant.* Model calibration
$INVESTINDIRECTTAXRT_{dr}$	*Indirect tax rate on investment commodity consumption.* Derived from base year SAM
$INVESTPARAM_{dr,i}$	*Parameter for scaling net return on capital.* Model calibration
$KNOWLANDTIME$	*Time prior to which actual land use change is known from historic data.*

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Table A.19 (continued)

Name	Description
	Set to t = 10
$KSFCONVERT_{dr,i}$	*Built capital stock-to-flow conversion parameter.* Derived from base year estimates of stock values by industry (refer to input data for $iBuiltcapital_{dr,i}$) and factor payments from the base year SAM
$LABFORCEADJUST_{sr}$	*Adjustment to the working age population to reflect the difference between StatsNZ "Working Age Population" and the total population aged 15-84.* Derived from the average difference between StatsNZ "Working Age Population" and the total population aged 15-84 for the years 2006-2018.
$LNDREVENUESH_{ft}$	*Share of returns to capital allocated to land.* Derived from base year SAM for industry in which farm type belongs
$LSFCONVERT_{dr}$	*Labour stock-to-flow conversion parameter.* Derived from base year estimates of labour supply (refer to input data for $iIndlaboursup_{sr}$) and labour factor payments from the base year SAM
$MAPTMITFF_{mt}$	*Vector identifying which mitigation state is the farm forestry mitigation.* All mitigation states have value of zero except $Miti09$ (farm forestry mitigation) which has a value of 1
$MAPTOAGIND_{ft,agi}$	*Mapping of Farm Types to Agricultural Industries - value of one indicates Farm Type belongs within Agricultural Industry, value of zero indicates does not belong.* $FaTyp01$-$FaTyp08$ mapped to $AgIn01$, $FaTyp09$-$FaTyp13$ mapped to $AgIn02$, $FaTyp14$ mapped to $AgIn03$, $FaTyp15$ mapped to $AgIn04$, $FaTyp16$ mapped to $AgIn06$, $FaTyp17$ mapped to $AgIn05$
$MAPTOIOAG_{IOag,ft}$	*Mapping of Farm Types to IOAgricultural Industries - value of one indicates Farm Type belongs within IOAgricultural Industry, value of zero indicates does not belong.* $FaTyp01$-$FaTyp13$ and $FaTyp16$-$FaTyp17$ mapped to $IOAg01$, $FaTyp14$ mapped to $IOAg02$, $FaTyp15$ mapped to $IOAg03$
$MAPTOMANGSYST_{ez,ft,mofa}$	*For each economic zone, mapping of Case Study Farms to Farm Types - value of one indicates Case Study Farm belongs to Farm Type, value of zero indicates does not belong.* The relevant dairy Case Study Farm for each economic zone (one from $MoFa45 - MoFa54$) is mapped to FaTyp14, for each economic zone $MoFa54$, $MoFa56$ and $MoFa57$ are mapped to $FaTyp16$, $FaTyp17$ and $FaTyp15$ respectfully, the mapping of drystock case study farms ($MoFa01$ - $MoFa44$) derived from industry expert's farm weighting exercise
$MAXCHANGERT_{ft}$	*Maximum proportion of land within a land use that can undergo land use change during any given year.*

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Table A.19 (continued)

Name	Description
	Set at 3.5% based on examination of historic land use change
$MAXREALLOCATERT$	*Maximum rate of reallocation of labour.* Set at 10% and derived from model calibration.
$MECRATIO_{sr}$	*Modified employment count (MEC) per Employed person.* Calculated from the regional ratio of MECs from the base year SAM and the base year Employment figures from Statistics NZ.
$MECTRANSOUT_{sr}$	*The proportion of labour force that stays in the same region or crosses regional boundaries to go to work.* Derived from base year linked Employer-Employee data.
$MOBILESH_{dr,i}$	*Share of investment that is mobile between industries.* Assumed to be 0.7 but can be adjusted in calibration
$MODFARMDEPCOEFHA_{mofa,mt}$	*Depreciation coefficients per hectare for each Case Study Farm and mitigation state.* Derived from Case Study Farm financials
$MODFARMINDTAXCOEFHA_{mofa,mt}$	*Tax coefficient per hectare for each Case Study Farm and mitigation state.* Derived from Case Study Farm financials
$MODFARMINPUTCOEFHA_{mofa,mt,c}$	*Commodity input coefficients per hectare for each Case Study Farm and mitigation state.* Derived from Case Study Farm financials
$MODFARMLABOURCOEFHA_{mofa,mt}$	*Labour input coefficient per hectare for each Case Study Farm and mitigation state.* Derived from Case Study Farm financials
$MODFARMOUTPUTCOEFHA_{mofa,mt,c}$	*Commodity output coefficients per hectare for each Case Study Farm and mitigation state.* Derived from Case Study Farm financials
$NATCAPCONVERT_{dr,i}$	*Natural capital stock-to-flow conversion parameter.* Derived from base year estimates of natural capital (refer to input data for $iNaturalcapital_{dr,nct}$) and natural capital payments calculated for the base year
$NZINTERESTWEIGHT$	*Weight given to the NZ interest rate in determining the value of savings from the rest of the world.* Estimated from linear regression of historic data
$PRODSCALAR_{dr,i}$	*Scalar to adjust industry production to account for indirect taxes.* Derived from base year SAM
$RAILMAP_{c}$	*Concordance defining rail freight service commodity.* For default 27 commodity model, all commodities set to 0 except commodity 20 which is set to one

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Table A.19 (continued)

Name	Description
$RDEP_{dr,i}$	*Rate of capital depreciation.* Derived from base capital stock estimates (refer to input data for $iBuiltcapital_{dr,i}$) and region and industry
$REGINVESTCONST_{dr,i}$	*Adjustment term in allocation of national investment to regions.* Derived from base year SAM
$REPORTINDMAP_{i,ri}$	*Mapping between Industries and Reporting Industries, value of 1 indicates Industry is included in Reporting Industry and value of zero indicates not included.* $Ind1$ maps to $ShBfDr$, $Ind2$ maps to $DaCaFm$, $Ind3$ maps to $FoLog$, $Ind4$ maps to $OtPrim$, $Ind5$ and $Ind7$ map to $OtFoManuf$, $Ind6$ maps to $DaManuf$, $Ind8$-$Ind10$ map to $OtManuf$, $Ind11$-$Ind15$ map to $UtCoTr$, $Ind13$ and $Ind14$ map to $TrHosp$, $Ind16$ and $Ind17$ map to $FiInReBu$, $Ind18$ and $Ind19$ map to $OtServ$
$RESIDUALBCAPITAL_{IOag}$	*Additional capital payments above that captured by Case Study Farms.* Derived from comparing sum of Case Study Farms with base SAM
$RESIDUALINPUT_{IOag,c}$	*Additional commodity inputs above that captured by Case Study Farms.* Derived from comparing sum of Case Study Farms with base SAM
$RESIDUALLABOUR_{IOag}$	*Additional labour payments above that captured by Case Study Farms.* Derived from comparing sum of Case Study Farms with base SAM
$RESIDUALLAND_{IOag}$	*Additional payments for land above that captured by Case Study Farms.* Derived from comparing sum of Case Study Farms with base SAM
$RESIDUALOUTPUT_{IOag,c}$	*Additional commodity outputs above that captured by Case Study Farms.* Derived from comparing sum of Case Study Farms with base SAM
$RETURNSCALE_{ez,agi}$	*Scalar applied to returns per hectare for primary industries prior to calculating land use change.* Set to 1 by default
$ROADMAP_{c}$	*Concordance defining road freight service commodity.* All commodities set to 0 except commodity 23 which is set to one
$RWDIRECTTAXRT_{dr}$	*Direct tax rate for the rest of the world.* Derived from base year SAM
$RWENTTRANSBS_{dr}$	*Transfers from the rest of the world to enterprises during the base year.* Derived from base year SAM
$RWFACTRT_{h,dr}$	*Share of factors supplied by the rest of the world.* Derived from base year SAM
$RWHHLDTRANSBS_{dr}$	*Transfers from the rest of the world to households during e base year.* Derived from base year SAM

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Table A.19 (continued)

Name	Description
$RWINDIRECTTAXRT_{dr}$	*Indirect tax rate on rest of world commodity consumption.* Derived from base year SAM
$RWREGSAVCONST_{dr}$	*Constant term in function that splits rest of world savings among regions.* Derived from base year SAM
$RWSAVCONST$	*Rest of world savings held constant.* Estimated from linear regression of historic data
$SAVREGTRANSBS_{dr}$	*Savings transferred out of the region during the base year.* Derived from base year SAM
$SETINVESTCQ_{dr,c}$	*Quantity of investment commodity consumption held constant.* Set equal to base year investment commodity consumption if this is less than zero, otherwise set to zero
$SHOCKINITIATION$	*Time at beginning of disruption (shock).* Set specifically for each scenario
$STOCKPILECOMM_{c}$	*Vector that identifies which commodities can be easily stockpiled value of 1 for commodities easily stockpiled, value of zero for commodities not easily stockpiled.* Assigned based on the nature of each commodity
$TASHAREEMP_{ta,i}$	*Share of regional industry employment within each Territorial Local Authority.* Calculated from data on Modified Employment Counts by Meshblock for 2017, Sourced from Market Economics BD Deluxe Database
$TAXHHLD_{dr}$	*Direct tax rate on households.* Derived from base year SAM
$TAZONEMAP_{ta,ez,agi}$	*For each agricultural industry, specifies the proportion of an economic zone within a Territorial Local Authority.* Estimated from quantities of effective hectares in Environment Southland's Land Use Map for 2015
$\alpha^{exchangert}$	*Exchange rate change parameter.* Derived from model calibration
$\alpha^{pbuiltcap}$	*Built capital price change parameter.* Derived from model calibration
$\alpha^{pexpcomm}$	*Export commodity price change parameter.* Derived from model calibration
α^{plab}	*Labour price change parameter.* Derived from model calibration
$\alpha^{pnatcap}_{nct}$	*Natural capital price change parameter.*

(continued on next page)

Table A.19 (continued)

Name	Description
	Derived from model calibration
$\alpha_{c}^{pregdomcomm}$	*Price of regional domestic commodities change parameter.* Derived from model calibration
δ_{dr}^{cc}	*Share parameter for capital types within the Constant Elasticity of Substitution (CES) function for composite capital.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{dr,i,c}^{cominput}$	*Share parameter for commodities within the CES function for composite intermediate inputs.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{dr,c}^{commddom}$	*Share parameter for domestic commodities within the CES function for composite domestic and imported commodities.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{dr,c}^{commdimp}$	*Share parameter for imported commodities within the CES function for composite domestic and imported commodities.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{sr,dr,c}^{commregd}$	*Share parameter for regional commodities within the CES function for composite domestic commodities.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{h,dr,i}^{fact}$	*Share parameter for factors within the CES function for composite factors.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{input,dr,i}^{fi}$	*Share parameter for factors/intermediate inputs within the CES function for composite inputs.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{ez,agi}^{fland}$	*Share parameter for agricultural industries within the CET function for land use change allocation.* Derived from calibration
$\delta_{g,dr,c}^{govtc}$	*Share parameter for commodities within the CES function for composite government consumption.* Calculated from base year SAM given assumed elasticity of substitution
$\delta_{dr,c}^{hhldc}$	*Share parameter for commodities within the CES function for composite household consumption.*

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Table A.19 (continued)

Name	Description
	Calculated from base year SAM given assumed elasticity of substitution
$\delta^{investc}_{dr,c}$	*Share parameter for commodities within the CES function for composite investment consumption.* Calculated from base year SAM given assumed elasticity of substitution
$\delta^{natcap}_{dr,i,nct}$	*Share parameter for natural capital types within the CES function for composite natural capital.* Calculated from base year SAM given assumed elasticity of substitution
$\epsilon^{cc}_{dr,i}$	*Elasticity of substitution between capital types.* Set to 0.1 for energy industries and for all other industries set to 0.3 (informed by Rae and Strutt (2005) and Hertel *et al.* (2012))
$\epsilon^{com}_{dr,c}$	*Elasticity of substitution between imported and domestic commodities.* Derived from Robson (2012, p.95) and Hertel *et al.* (2012)
$\epsilon^{cominput}_{dr,i}$	*Elasticity of substitution between commodities of different types.* Set to 0.8 for all industries
$\epsilon^{fact}_{dr,i}$	*Elasticity of substitution between factors.* Derived from Hertel *et al.* (2012)
$\epsilon^{factint}_{dr,i}$	*Elasticity of substitution between factors and intermediate inputs.* Set to 0.5 for all industries (Robson, 2012)
ϵ^{fland}_{ez}	*Elasticity of substitution between primary industry land uses.* Derived from calibration
$\epsilon^{govtc}_{g,dr}$	*Elasticity of substitution between commodities in government consumption.* Set to 0.5
ϵ^{hhldc}_{dr}	*Elasticity of substitution between commodities in household consumption.* Set to 0.5 (Robson, 2012)
$\epsilon^{investc}_{dr}$	*Elasticity of substitution between commodities in investment consumption.* Set to 0.5
$\epsilon^{natcap}_{dr,i}$	*Elasticity of natural capital substitution.* Set to 0.4 for forestry and logging and to 20 for other agriculture industries (Rae and Strutt, 2011)
$\epsilon^{regcom}_{dr,c}$	*Elasticity of substitution between commodities from different regions.* Derived from Robson (2012)
γ^{cc}_{dr}	*Scale parameter for the CES function for composite capital.*

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Table A.19 (continued)

Name	Description
	Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr,i}^{cominput}$	*Scale parameter for the CES function for composite intermediate inputs.* Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr,c}^{commd}$	*Scale parameter for the CES function for composite domestic and imported commodities.* Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr,c}^{commregd}$	*Scale parameter for the CES function for composite domestic commodities.* Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr,i}^{fact}$	*Scale parameter for the CES function for composite factors.* Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr,i}^{fi}$	*Scale parameter for the CES function for composite factors and intermediate inputs.* Calculated from base year SAM given assumed elasticity of substitution
γ_{ez}^{fland}	*Share parameter for Constant Elasticity of Transformation (CET) function for land use change allocation.* Derived from calibration
$\gamma_{g,dr}^{govtc}$	*Scale parameter for the CES function for composite government consumption.* Calculated from base year SAM given assumed elasticity of substitution
γ_{dr}^{hhldc}	*Scale parameter for the CES function for composite household consumption.* Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr}^{investc}$	*Scale parameter for the CES function for composite investment consumption.* Calculated from base year SAM given assumed elasticity of substitution
$\gamma_{dr,i}^{natcap}$	*Scale parameter for the CES function for composite natural capital.* Calculated from base year SAM given assumed elasticity of substitution
$\psi_{sr,c}^{com}$	*Elasticity of transformation between export and domestic commodity supply.*

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Table A.19 (continued)

Name	Description
	For each commodity, elasticities are assumed to be similar to that assigned for substitution between imports and domestic goods, i.e. $\epsilon^{com}_{dr,c}$
$\psi^{comsup}_{sr,i}$	*Elasticity of transformation in the supply of different types of commodities by industries.* Set to 0.8
$\psi^{natcap}_{dr,nct}$	*Elasticity of transformation in the supply of natural capital to different industries.* Set to 0.4 (informed by Rae and Strutt (2011))
$\psi^{regcom}_{sr,c}$	*Elasticity of transformation in the supply of commodities to different regions.* For each commodity, set equal to the elasticity of substitution between different region's goods, $\epsilon^{regcom}_{dr,c}$
$\theta^{commregs}_{sr,c}$	*Scale parameter for the CET function for supply of commodities to different regions.* Calculated from base year SAM given assumed elasticity of transformation
$\theta^{commsdexp}_{sr,c}$	*Scale parameter for the CET function for supply of commodities to either the domestic or export market.* Calculated from base year SAM given assumed elasticity of transformation
$\theta^{comsup}_{sr,i}$	*Scale parameter for the CET function for the supply of different types of commodities by industries.* Calculated from base year SAM given assumed elasticity of transformation
$\theta^{natcap}_{dr,nct}$	*Scale parameter for the CET function for the supply of natural capital to different industries.* Calculated from base year SAM given assumed elasticity of transformation
$\xi^{commregs}_{sr,dr,c}$	*Share parameter for regions in the CET function for the supply of domestic commodities to different NZ regions.* Calculated from base year SAM given assumed elasticity of transformation
$\xi^{commsdom}_{sr,c}$	*Share parameter for the domestic market in the CET function for the supply of commodities to the domestic or export market.* Calculated from base year SAM given assumed elasticity of transformation
$\xi^{commsexp}_{sr,c}$	*Share parameter for the export market in the CET function for the supply of commodities to the domestic or export market.* Calculated from base year SAM given assumed elasticity of transformation

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Table A.19 (continued)

Name	Description
$\xi^{comsup}_{sr,i,c}$	*Share parameter for commodities in the CET function for the supply of different types of commodities by industries.* Calculated from base year SAM given assumed elasticity of transformation
$\xi^{natcap}_{dr,i,nct}$	*Share parameter for industries in the CET function for the supply of natural capital to different industries.* Calculated from base year proportions of Natural Capital between different industries and regions given assumed elasticity of transformation
$\tau_{casurplus}$	*Time to adjust perceived current account surplus to actual current account surplus.* Derived from model calibration
τ_{income}	*Time to adjust recognised income to actual income.* Derived from model calibration
$\tau_{industry}$	*Time to adjust desired industry production to value of industry demand.* Derived from model calibration
$\tau_{interest}$	*Time to adjust interest rate to desired interest rate.* Derived from model calibration
τ_{prices}	*Time to adjust price stocks to calculated prices.* Set to the time step Δt.
τ	*Time adjustment constant for stocks that we wish to adjust almost instantaneously.* Set to the time step Δt.

Table A.20 Description of agriculture policy interface exogenous inputs

Name	Description
$ANNUALFENCE_{fmu,agi}$	*Annual fencing costs over period during which fencing undertaken, in base year dollar terms*
$ANPCONSENTC_{agi}$	*Average annual costs of Council consenting for farm plans, excludes first plan year, in base year dollar terms*
$ANPOTHERC_{agi}$	*Average annual other costs for farm plans, excludes first plan year, in base year dollar terms*
$DATENONPERMITTED_{ez,ft,mt}$	*Date by which all adoptions out of a mitigation state must be complete*

(continued on next page)

Table A.20 (continued)

Name	Description
$FARMFRINPUTCOEFHA_c$	*Farm forestry input coefficient per hectare*
$FARMFROUTPUTCOEFHA_c$	*Farm forestry output coefficient per hectare*
$FARMRETIREMENTBYTIME_{ez,agi}(t)$	*Quantity of effective hectares retired from production*
$FENCEEND_{fmu,agi}$	*Time fencing ends*
$FENCEMAP_c$	*Vector identifying fencing commodity - set to zero for all commodities except commodity 21 set to one*
$FENCESTART_{fmu,agi}$	*Time fencing commences*
$FIRSTPCONSENTC_{agi}$	*Council consent costs for farm plans, first year plan only, in base year dollar terms*
$FIRSTPOTHERC_{agi}$	*Other costs for farm plans, first year plan only, in base year dollar terms*
$FIRSTPTIME_{agi}$	*Time of first farm plan*
$FRRETIRE_{ez,agi}$	*Land retirement within farms to farm forestry (hectares per effective ha)*
$INTENSITYSCALE_{ez,ft}$	*De-intensification scalar associated with farm forestry mitigation*
$LANDUSERESTRICTION_{ez,ft,mt}$	*Maximum area of effective hectares allowed*
$LRIMPEND_{ez,agi}$	*Implementation end time for land retirement within farms*
$LRIMPSTART_{ez,agi}$	*Implementation start time for land retirement within farms*
$MAXIMUMRATE_{ez,ft,mt}$	*Maximum adoption rate*
$NONPERMITNOTICEDATE_{ez,ft,mt}$	*Date at which adoptions out of a mitigation state commence*

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Table A.20 (continued)

Name	Description
$NPRETIRE_{ez,agi}$	*Land retirement within farms to non-productive use (hectares per effective hectares)*
$PLANMAP1_c$	*Vector of zeros except for the commodity type associated with council consenting which is allocated a value of one*
$PLANMAP2_c$	*Vector of zeros except for the commodity type associated with farm plan creation which is allocated a value of one*
$SELECTEDADOPTION_{ez,ft,mt}$	*Adoption rate selected (1=early adoption, 2=medium adoption, 3=late adoption)*
$VIABLEMIT_{ez,ft}$	*Vector indicating whether farm forestry mitigation is a viable mitigation (1=yes, 0=no)*

Table A.21 Description of municipal exogenous inputs & auxiliaries

Name	Description
$CAPINVESTRESP_{yrmt,mf,lt}$	*Share of funding responsibility for new capital*
$FISCOMMMAP_c$	*Vector identifying financial intermediate services commodity (value of one for com24, zero for all other commodities)*
$FISSHARE$	*Component of interest rate charged for financial intermediate services*
$LOANRATE$	*Interest rate for loan*
$LOANYR_{yrmt}$	*Year loan commences*
$MUNCAPINVESTBYTIME_{dr}(t)$	*Total capital investment for new municipal wastewater treatment (excludes land purchases)*
$MUNINDTAXRT$	*Industry tax rate for municipal wastewater treatment*

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Table A.21 (continued)

Name	Description
$MUNINVESTRATIO_c$	*Commodity share of capital investment for municipal wastewater treatment*
$MUNLANDINVESTBYTIME_{dr}(t)$	*Total land purchases for new municipal wastewater treatment*
$OPEXDEMANDBYTIME_{dr}(t)$	*Total operating expenditure on new municipal wastewater treatment*
$OPEXRESPBYTME_{mf}(t)$	*Share of funding responsibility for operating expenditures*
$PAYMENTSPERYEAR$	*Payments per year*
$WASTEMAP_c$	*Vector identifying wastewater treatment service*
$WASTEINDMAP_i$	*Vector identifying wastewater treatment industry*
$YEARSFORLOAN$	*Number of years for loan*

Table A.22 Description of time-varying exogeneous inputs

Name	Description
$ACINFLATIONRT(t)$	*Actual inflation rate* Historic data of the headline CPI (Reserve Bank NZ)
$ACRWSAVINGS(t)$	*Actual rest of world savings.* Historic data (Statistics NZ)
$ACTUALEXCHANGERT(t)$	*Actual exchange rate* Historic data (Reserve Bank NZ)
$ACTUALGDPGAP(t)$	*Actual GDP gap* Historic data (Reserve Bank NZ)
$ACTUALINTERESTRT(t)$	*Actual interest rate* Historic data of the 90 day bank bill yield rate (Reserve Bank NZ)
$ACTUALLAND_{ez,agi}(t)$	*Actual land use* Historic data (Environment Southland Land Use Map)
$ADDHHLDTRAVEL_{dr,c}(t)$	*Net additional household consumption of transport-related commodities.*

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Table A.22 (continued)

Name	Description
	Scenario specific settings from assessment of direct transport outage impacts
$ADVALOREMEXPORTP_c(t)$	*Ad valorem export price parameter* Scenario-specific percentage-based adjustment to the perceived price of New Zealand exports
$ADVALOREMIMPORTP_c(t)$	*Ad valorem import price parameter* Scenario-specific percentage-based adjustment to the perceived price experienced in New Zealand of imported commodities
$ADVALOREMIMPTARIFF_c(t)$	*Ad valorem import tariff parameter* Scenario-specific additional tariffs charged on imports (applied as a percentage increase in price)
$CAPINDEX_{dr,i}(t)$	*Capital augmenting index parameter* Scenario-specific adjustment to 'effective' capital factors held by each industry
$DMARGINSHOCKCOEF_{sr,dr,c}(t)$	*Net additional domestic transport margins per unit of commodity* Scenario specific settings from assessment of direct transport outage impacts
$EMARGINSHOCKCOEF_{sr,c,m}(t)$	*Net additional export transport margins per unit of commodity* Scenario specific settings from assessment of direct transport outage impacts
$HHLDTAXRTADJUST_{dr}(t)$	*Adjustment to household tax rate* Scenario-specific adjustment to household tax rate
$IMARGINSHOCKCOEF_{dr,c,m}(t)$	*Net additional import transport margins per unit of commodity* Scenario specific settings from assessment of direct transport outage impacts
$INDTAXRTADJUST_i(t)$	*Adjustment to industry tax rate* Scenario-specific adjustment to industry tax rate
$LABINDEX_{dr,i}(t)$	*Labour augmenting index parameter* Scenario-specific adjustment to 'effective' labour factors held by industries
$MFPADJUST_{dr,i}(t)$	*Adjustment to multifactor productivity.* Derived from model calibration
$MULTIFACTORPROD_{dr,i}(t)$	*Multifactor productivity index* Econometrically determined projection
$NATURALGDP(t)$	*Natural GDP* Potential output from Reserve Bank estimation with econometrically determined projection going forward
$NETTRANSFERS(t)$	*Net transfers*

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Table A.22 (continued)

Name	Description
	Historic time series of net rest of world transfers
$NONPRODINVESTSH(t)$	*Non-productive investment share* Scenario-specific share of investment assigned to non-productive items
$OPERABILITY_{sr,i}(t)$	*Industry operability index* Scenario specific settings from the Business Behaviours Module
$PARTICIPATIONRT_{sr}(t)$	*Workforce participation rate* Historic participation rates for New Zealand regions with forecast adjustments from 2019-22 (Treasury 2018) and held constant post 2022
$PCOMMWORLDEXP_{c}(t)$	*World commodity prices for exports* Econometrically determined projection
$PCOMMWORLDIMP_{c}(t)$	*World commodity prices for imports* Econometrically determined projection
$POPULATION_{dr}(t)$	*Total population in each region* Statistics New Zealand's regional population projections
$REGSHEXPMAR_{sr,dr,m}(t)$	*Regional supply share of export margins* Scenario specific settings
$REGSHIMPMAR_{sr,dr,m}(t)$	*Regional supply share of import margins* Scenario specific settings
$WORKINGAGEPOP_{sr}(t)$	*Working age population* Statistics New Zealand's regional population projections
$WORLDGDPINDX(t)$	*World gross domestic product index* Historic data and econometrically determined projection
$WORLDINTERESTRT(t)$	*World interest rate* Historic data and econometrically determined projection, capped at 4%

B Model equations

B.1 Household module equations

B.1.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Rhhldincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualhhldincome_{dr} - \mathbf{Rhhldincome}_{dr}\right) \tag{B.1}$$

$$\frac{d}{dt}\left(\mathbf{Phhldcc}_{dr}\right) = \frac{1}{\tau_{prices}}\left(actualphhldcc_{dr} - \mathbf{Phhldcc}_{dr}\right) \tag{B.2}$$

B.1.2 Auxiliaries

$$\begin{aligned} actualhhldincome_{dr} &= capincomehhld_{dr} + entincomehhld_{dr} + labincomesupply_{sr\rightarrow dr} + rwhhldtrans_{dr} \\ &+ \sum_{g}\left(govthhldtrans_{g,dr}\right) + hhldregtransout_{DReg1\leftrightarrow DReg2} - hhlddirecttax_{dr} - hhldloanpayments_{dr} \\ &- \sum_{c} vmunopex_{dr,c,mf=Hhld} \end{aligned} \tag{B.3}$$

$$capincomehhld_{dr} = caplocalhhldtrans_{dr} + capreghhldtrans_{DReg1\leftrightarrow DReg2} \tag{B.4}$$

$$entincomehhld_{dr} = enthhldtrans_{dr} + entreghhldtrans_{DReg1\leftrightarrow DReg2} \tag{B.5}$$

$$labincomesupply_{sr} = \sum_{dr}\left(\frac{labincomedregion_{dr}\, reglaboursupply_{sr,dr}}{\sum_{sr}\left(reglaboursupply_{sr,dr}\right)}\right) \tag{B.6}$$

$$labincomedregion_{dr} = \sum_{i}\left(factorsu_{h=LAB,dr,i}\left(1 - RWFACTRT_{h=LAB,dr}\right)\mathbf{Pfact}_{h=LAB,dr,i}\right) \tag{B.7}$$

$$\begin{aligned} hhldrwtrans_{dr} &= HHLDRWTRANSBS_{dr}\, WORLDGDPINDX(t)\left(1 - FCSHHHLDRWTRANS_{dr}\right) \\ &+ HHLDRWTRANSBS_{dr}\, WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHHHLDRWTRANS_{dr} \end{aligned} \tag{B.8}$$

$$\begin{aligned} rwhhldtrans_{dr} &= RWHHLDTRANSBS_{dr}\, WORLDGDPINDX(t)\left(1 - FCSHRWHHLDTRANS_{dr}\right) \\ &+ RWHHLDTRANSBS_{dr}\, WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right) FCSHRWHHLDTRANS_{dr} \end{aligned} \tag{B.9}$$

$$hhldenttrans_{dr} = (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \times HHLDENTTRANSRT_{dr} \quad \text{(B.10)}$$

$$hhldgovttrans_{g,dr} = (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \times HHLDGOVTTRANSRT_{g,dr} \quad \text{(B.11)}$$

$$hhldregtransout_{dr} = (\mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - addtravelcosts_{dr}) \times HHLDREGTRANSRT_{dr} \quad \text{(B.12)}$$

$$hhldtotal_{dr} = \mathbf{Rhhldincome}_{dr} - hhldrwtrans_{dr} - hhldregtransout_{dr} - hhldenttrans_{dr} - \sum_{g}(hhldgovttrans_{g,dr}) - addtravelcosts_{dr} \quad \text{(B.13)}$$

$$hhldsavings_{dr} = hhldtotal_{dr}\,(1 - hhldconsumprt_{dr}) + HHLDSAVINGADJUST_{dr} \quad \text{(B.14)}$$

$$totalhhldconsump_{dr} = hhldtotal_{dr}\, hhldconsumprt_{dr} \quad \text{(B.15)}$$

$$hhldconsumprt_{dr} = \left[\left(\frac{realinterestrt}{BASEREALINTERESTRT} - 1\right) CIRELASTICITY_{dr} + 1\right] \times BASECONSUMPRT_{dr} \quad \text{(B.16)}$$

$$actualphhldcc_{dr} = \frac{\sum_c (\mathbf{Pcompcommd}_{dr,c}\, hhldconsump_{dr,c})}{qhhldcc_{dr}} \quad \text{(B.17)}$$

$$qhhldcc_{dr} = \gamma_{dr}^{hhldc} \left[\sum_c \left(\delta_{dr,c}^{hhldc} (hhldconsump_{dr,c})^{\eta_{dr}^{hhldc}}\right)\right]^{\frac{1}{\eta_{dr}^{hhldc}}} \quad \text{(B.18)}$$

$$hhldconsump_{dr,c} = \left[(\gamma_{dr}^{hhldc})^{\eta_{dr}^{hhldc}}\, \delta_{dr,c}^{hhldc} \frac{\mathbf{Phhldcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta_{dr}^{hhldc}}} hhldcompcommd_{dr} \quad \text{(B.19)}$$

$$\eta_{dr}^{hhldc} = \frac{\epsilon_{dr}^{hhldc} - 1}{\epsilon_{dr}^{hhldc}} \quad \text{(B.20)}$$

$$hhldcompcommd_{dr} = \frac{totalhhldconsump_{dr} - hhldindirecttax_{dr}}{\mathbf{Phhldcc}_{dr}} \quad \text{(B.21)}$$

$$hhldindirecttax_{dr} = totalhhldconsump_{dr} \times hhldindtaxrtadjusted_{dr} \quad \text{(B.22)}$$

$$hhlddirecttax_{dr} = (capincomehhld_{dr} + labincomesupply_{sr\to dr}) \times TAXHHLD_{dr} \quad \text{(B.23)}$$

$$hhldindtaxrtadjusted_{dr} = HHLDINDTAXRT_{dr} + HHLDTAXRTADJUST_{dr}(t) \quad \text{(B.24)}$$

$$\begin{aligned} hhldloanpayments_{dr=DReg1} &= \textstyle\sum_{lt} totalloanpayments_{rt=Normal,mf=Hhld,lt} \\ hhldloanpayments_{dr=DReg2} &= 0 \end{aligned} \quad \text{(B.25)}$$

B.2 Government module equations

B.2.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Rgovtincome}_{g,dr}\right) = \frac{1}{\tau_{income}}\left(govtincome_{g,dr} - \mathbf{Rgovtincome}_{g,dr}\right) \tag{B.26}$$

$$\frac{d}{dt}\left(\mathbf{Pgovtcc}_{g,dr}\right) = \frac{1}{\tau_{prices}}\left(actualpgovtcc_{g,dr} - \mathbf{Pgovtcc}_{g,dr}\right) \tag{B.27}$$

B.2.2 Auxiliaries

$$\begin{aligned} govtincome_{g,dr} =& directtaxincome_{g,dr} + indirecttaxincome_{g,dr} + capgovttrans_{g,dr} + entgovttrans_{g,dr} \\ &+ hhldgovttrans_{g,dr} + \sum_{sr} betwgovttransin_{g,sr,dr} - govtdirecttax_{g,dr} \\ &- centralgovmunpays_{g,dr} + importtariffs_{g,dr} \end{aligned} \tag{B.28}$$

$$\begin{aligned} directtaxincome_{g,dr} = &\left[entdirecttax_{dr} + hhlddirecttax_{dr} + rwdirecttax_{dr} + \sum_{g}\left(govtdirecttax_{g,dr}\right)\right] \\ &\times DIRECTTAXSH_{g,dr} \end{aligned} \tag{B.29}$$

$$\begin{aligned} indirecttaxincome_{g,dr} = &\Bigg[investindirecttax_{dr} + rwindirecttax_{dr} + hhldindirecttax_{dr} \\ &\sum_{i}(indindirecttax_{dr,i}) + \sum_{g}(govtindirecttax_{g,dr})\Bigg] \times INDIRECTTAXSH_{g,dr} \end{aligned} \tag{B.30}$$

$$govtdirecttax_{g,dr} = \left[capgovttrans_{g,dr} + entgovttrans_{g,dr}\right] \times GOVTDIRECTTAXRT_{g,dr} \tag{B.31}$$

$$\begin{aligned} govtrwtrans_{g,dr} =& \operatorname{sgn}\left(\mathbf{Casurplus}\right)\left|\mathbf{Casurplus}\right|^{EGOVTTRANS}\left(\frac{GOVTRWTRANSBS_{g,dr}}{\sum_g \sum_{dr}\left(GOVTRWTRANSBS_{g,dr}\right)}\right) \\ &+ GOVTRWTRANSBS_{g,dr} \end{aligned} \tag{B.32}$$

$$betwgovttransin_{g,dr} = betwgovttransout_{CentralG \leftrightarrow LocalG,dr} \tag{B.33}$$

$$betwgovttransout_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times BTWGOVTTRANSRT_{g,dr} \tag{B.34}$$

$$govthhldtrans_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTHHLDTRANSRT_{g,dr} \tag{B.35}$$

$$govtsavings_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTSAVRT_{g,dr} \tag{B.36}$$

$$totalgovtconsump_{g,dr} = \left[\mathbf{Rgovtincome}_{g,dr} - govtrwtrans_{g,dr}\right] \times GOVTCONSUMPRT_{g,dr} \tag{B.37}$$

$$actualpgovtcc_{g,dr} = \frac{\sum_c \left(govtconsump_{g,dr,c}\,\mathbf{Pcompcommd}_{dr,c}\right)}{qgovtcc_{g,dr}} \tag{B.38}$$

$$qgovtcc_{g,dr} = \gamma^{govtc}_{g,dr}\left[\sum_c \left(\delta^{govtc}_{g,dr,c}\left(govtconsump_{g,dr,c}\right)^{\eta^{govtc}_{g,dr}}\right)\right]^{\frac{1}{\eta^{govtc}_{g,dr}}} \tag{B.39}$$

$$govtconsump_{g,dr,c} = \left[\left(\gamma^{govtc}_{g,dr}\right)^{\eta^{govtc}_{g,dr}}\,\delta^{govtc}_{g,dr,c}\frac{\mathbf{Pgovtcc}_{g,dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta^{govtc}_{g,dr}}}\,govtcompcommd_{g,dr} \tag{B.40}$$

$$\eta^{govtc}_{g,dr} = \frac{\epsilon^{govtc}_{g,dr} - 1}{\epsilon^{govtc}_{g,dr}} \tag{B.41}$$

$$govtcompcommd_{g,dr} = \frac{totalgovtconsump_{g,dr} - govtindirecttax_{g,dr}}{\mathbf{Pgovtcc}_{g,dr}} \tag{B.42}$$

$$govtindirecttax_{g,dr} = totalgovtconsump_{g,dr}\,GOVTINDIRECTTAXRT_{g,dr} \tag{B.43}$$

$$\begin{aligned} centralgovmunpays_{g,dr} = &\left(\sum_{lt} totalloanpayments_{rt=Normal,mf=Cgovt,lt} + \sum_c vmunopex_{dr,c,mf=Cgovt}\right)\\ &\times \frac{\mathbf{Rgovtincome}_{g,dr}}{\sum_{dr}\mathbf{Rgovtincome}_{g,dr}} \end{aligned} \tag{B.44}$$

B.3 Enterprise module equations

B.3.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Renterincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(actualenterincome_{dr} - \mathbf{Renterincome}_{dr}\right) \tag{B.45}$$

B.3.2 Auxiliaries

$$\begin{aligned} actualenterincome_{dr} = &capentertrans_{dr} + hhldenttrans_{dr} + rwenttrans_{dr}\\ &+ entregtransout_{DReg1\leftrightarrow DReg2} - entdirecttax_{dr} \end{aligned} \tag{B.46}$$

$$entdirecttax_{dr} = \left(capentertrans_{dr} + rwenttrans_{dr}\right) \times ENTTAXRT_{dr} \tag{B.47}$$

$$\begin{aligned} rwenttrans_{dr} = &RWENTTRANSBS_{dr}\,WORLDGDPINDX(t)(1 - FCSHRWENTTRANS_{dr})\\ &+RWENTTRANSBS_{dr}\,WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right)FCSHRWENTTRANS_{dr} \end{aligned} \tag{B.48}$$

$$\begin{aligned} entrwtrans_{dr} = &ERWTRANSBS_{dr}\,WORLDGDPINDX(t)(1 - FCSHENTRWTRANS)\\ &+ERWTRANSBS_{dr}\,WORLDGDPINDX(t)\left(\frac{1}{\mathbf{Exchangert}}\right)FCSHENTRWTRANS \end{aligned} \tag{B.49}$$

$$entregtransout_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EREGTRANSRT_{dr} \tag{B.50}$$

$$entgovttrans_{g,dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EGOVTTRANSRT_{g,dr} \tag{B.51}$$

$$enthhldtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times EHHLDTRANSRT_{dr} \tag{B.52}$$

$$entreghhldtrans_{dr} = [\mathbf{Renterincome}_{dr} - entrwtrans_{dr}] \times ERHHLDTRANSRT_{dr} \tag{B.53}$$

$$entsavtrans_{dr} = \left[\mathbf{Renterincome}_{dr} - entrwtrans_{dr}\right] \times ESAVTRANSRT_{dr} \tag{B.54}$$

B.4 Industry module equations

B.4.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Desiredprod}_{dr,i}\right) = \frac{1}{\tau_{industry}}\left(\sum_{c}\left(vinddemandc_{sr\rightarrow dr,i,c}\right) - \mathbf{Desiredprod}_{dr,i}\right) \tag{B.55}$$

$$\frac{d}{dt}\left(\mathbf{Industryaccount}_{dr,i}\right) = industryinc_{dr,i} - indexpendu_{dr,i} \tag{B.56}$$

$$\frac{d}{dt}\left(\mathbf{Industrybalance}_{dr,i}\right) = \frac{1}{\tau}\left(realindustrybalance_{dr,i} - \mathbf{Industrybalance}_{dr,i}\right) \tag{B.57}$$

B.4.2 Auxiliaries

$$vinddemandc_{sr,i,c} = supcoeff_{sr,i,c}\, vcomdemand_{sr,c} \tag{B.58}$$

$$\begin{aligned} vcomdemand_{sr,c} = & \sum_{dr}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\, regdomcommd_{sr,dr,c}\right) \\ & + expcommodityd_{sr,c}\frac{\mathbf{Pexpcomm}_{sr,c}}{\mathbf{Exchangert}} \end{aligned} \tag{B.59}$$

$$actualprod_{dr,i} = \begin{cases} maxprodsup_{dr,i} & \text{if} \quad OPERABILITY_{sr\rightarrow dr,i}(t) = 1 \\ \min[maxprodsup_{dr,i}, maxprod_{dr,i}] & \text{if} \quad OPERABILITY_{sr\rightarrow dr,i}(t) < 1 \end{cases} \tag{B.60}$$

$$maxprodsup_{dr,i} = Desiredprod_{dr,i} \tag{B.61}$$

$$\begin{aligned} realindustrybalance_{dr=DReg1,i} &= industryinc_{dr=DReg1,i} - indexpendu_{dr=DReg1,i} \\ &\quad - \textstyle\sum_{fmu}(indfencingcosts_{fmu,i} + indplancosts_{fmu,i}) \\ realindustrybalance_{dr=DReg2,i} &= industryinc_{dr=DReg2,i} - indexpendu_{dr=DReg2,i} \end{aligned} \tag{B.62}$$

$$indexpendu_{dr,i} = \sum_{c} (domcommexpend_{dr,i,c} + importcommexpend_{dr,i,c}) + \sum_{h} (factorsu_{h,dr,i}\, \mathbf{Pfact}_{h,dr,i}) + indindirecttax_{dr,i} \tag{B.63}$$

$$importcommexpend_{dr,i,c} = induseshare_{dr,i,c}\, importdemand_{dr,c}\, pimpcommnz_{dr,c} \tag{B.64}$$

$$domcommexpend_{dr,i,c} = induseshare_{dr,i,c} \sum_{sr} (domcommodityuse_{sr,dr,c}\, pregdomcomminclmargin_{sr,dr,c}) \tag{B.65}$$

$$induseshare_{dr,i,c} = \frac{indconsump_{dr,i,c}}{totalcomdemand_{dr,c}} \tag{B.66}$$

$$domcommodityuse_{dr,i,c} = \min[regcdomcomms_{sr,dr,c}, regcdomcommd_{sr,dr,c}] \tag{B.67}$$

$$industryinc_{dr,i} = \sum_{c} (actualsupply_{sr \to dr,i,c}) \tag{B.68}$$

$$actualsupply_{sr,i,c} = \min[potentialsales_{sr,i,c}, vinddemandc_{sr,i,c}] \tag{B.69}$$

$$potentialsales_{sr,i,c} = indcommoditys_{sr,i,c}\, \mathbf{Pcompcomms}_{sr,c} \tag{B.70}$$

$$compfactord_{dr,i} = \frac{maxprodsup_{dr,i}}{unitcost_{dr,i}} \frac{factinputshare_{dr,i}}{multifactorprod2_{dr,i}} \tag{B.71}$$

$$effectcompfactord_{dr,i} = \frac{actualprod_{dr,i}}{unitcost_{dr,i}} \frac{factinputshare_{dr,i}}{multifactorprod2_{dr,i}} \tag{B.72}$$

$$unitcost_{dr,i} = interinputunitcost_{dr,i} + factinputunitcost_{dr,i} \tag{B.73}$$

$$interinputunitcost_{dr,i} = \left(interinputshare_{dr,i}\, \mathbf{Pintinputs}_{dr,i}\, (1 + indindirecttaxrtadjusted_{dr,i})\right) \times \left(\frac{1}{multifactorprod2_{dr,i}}\right) \tag{B.74}$$

$$factinputunitcost_{dr,i} = \left(factinputshare_{dr,i}\, \mathbf{Pcfact}_{dr,i}\right) \times \left(\frac{1}{multifactorprod2_{dr,i}}\right) \tag{B.75}$$

$$indindirecttax_{dr,i} = \left[\sum_{c} (indconsump_{dr,i,c}\, \mathbf{Pcompdomcommd}_{dr,c})\right] \times indindirecttaxrtadjusted_{dr,i} \tag{B.76}$$

$$indindirecttaxrtadjusted_{dr,i} = INDINDIRECTTAXRT_{dr,i} + INDTAXRTADJUST_{i}(t) \tag{B.77}$$

B.5 Commodities module equations

B.5.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Estimports}_{dr,c}\right) = \left(importdemand_{dr,c} - \mathbf{Estimports}_{dr,c}\right) \tag{B.78}$$

$$\frac{d}{dt}\left(\mathbf{Pcindustrys}_{sr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcindustrys_{sr,i} - \mathbf{Pcindustrys}_{sr,i}\right) \tag{B.79}$$

$$\frac{d}{dt}\left(\mathbf{Pcompcommd}_{dr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpccd_{dr,c} - \mathbf{Pcompcommd}_{dr,c}\right) \tag{B.80}$$

$$\frac{d}{dt}\left(\mathbf{Pcompcomms}_{sr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpccs_{sr,c} - \mathbf{Pcompcomms}_{sr,c}\right) \tag{B.81}$$

$$\frac{d}{dt}\left(\mathbf{Pcompdomcommd}_{dr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpcdcd_{dr,c} - \mathbf{Pcompdomcommd}_{dr,c}\right) \tag{B.82}$$

$$\frac{d}{dt}\left(\mathbf{Pcompdomcomms}_{sr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpcdcs_{sr,c} - \mathbf{Pcompdomcomms}_{sr,c}\right) \tag{B.83}$$

$$\frac{d}{dt}\left(\mathbf{Pexpcomm}_{sr,c}\right) = \left(\left(\frac{1}{exportratio_{sr,c}}\right)^{\alpha^{pexpcomm}} - 1\right)\mathbf{Pexpcomm}_{sr,c} \tag{B.84}$$

$$\frac{d}{dt}\left(\mathbf{Pfiinputs}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpfiinputs_{dr,i} - \mathbf{Pfiinputs}_{dr,i}\right) \tag{B.85}$$

$$\frac{d}{dt}\left(\mathbf{Pintinputs}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpintinputs_{dr,i} - \mathbf{Pintinputs}_{dr,i}\right) \tag{B.86}$$

$$\frac{d}{dt}\left(\mathbf{Pregdomcomm}_{sr,dr,c}\right) = \left(\left(\frac{1}{excessproduction_{sr,dr,c}}\right)^{\alpha_c^{pregdomcomm}} - 1\right)\mathbf{Pregdomcomm}_{sr,dr,c} \tag{B.87}$$

$$\frac{d}{dt}\left(\mathbf{Pperceivedcompcommd}_{dr,c}\right) = \frac{1}{\tau_{prices}}\left(actualpercccd_{dr,c} - \mathbf{Pperceivedcompcommd}_{dr,c}\right) \tag{B.88}$$

B.5.2 Auxiliaries

$$exportratio_{sr,c} = \frac{expcommoditys_{sr,c}}{expcommodityd_{sr,c}} \tag{B.89}$$

$$expcommoditys_{sr,c} = \left[\left(\theta_{sr,c}^{commsdexp}\right)^{\phi_{sr,c}^{com}} \xi_{sr,c}^{commsexp}\frac{\mathbf{Pcompcomms}_{sr,c}}{pexpcommnz_{sr,c}}\right]^{\frac{1}{1-\phi_{sr,c}^{com}}} regcommoditys_{sr,c} \tag{B.90}$$

$$domcommoditys_{sr,c} = \left[(\theta^{commsdexp}_{sr,c})^{\phi^{com}_{sr,c}} \xi^{commsdom}_{sr,c} \frac{\mathbf{Pcompcomms}_{sr,c}}{\mathbf{Pcompdomcomms}_{sr,c}} \right]^{\frac{1}{1-\phi^{com}_{sr,c}}} regcommoditys_{sr,c} \tag{B.91}$$

$$pexpcommnz_{sr,c} = \mathbf{Pexpcomm}_{sr,c} \left(\frac{1}{\mathbf{Exchangert}} \right) \tag{B.92}$$

$$regcommoditys_{sr,c} = \sum_i (indcommoditys_{sr,i,c}) \tag{B.93}$$

$$indcommoditys_{sr,i,c} = \left[(scalecommsup1_{sr,i})^{\phi^{comsup}_{sr,i}} sharecommsup1_{sr,i,c} \frac{\mathbf{Pcindustrys}_{sr,i}}{\mathbf{Pcompcomms}_{sr,c}} \right]^{\frac{1}{1-\phi^{comsup}_{sr,i}}} \times regindprodincltax_{sr,i} \tag{B.94}$$

$$supcoeff_{sr,i,c} = \frac{indcommoditys_{sr,i,c}}{\sum_i (indcommoditys_{sr,i,c})} \tag{B.95}$$

$$regindprodincltax_{sr,i} = \frac{effectcompfactoru_{dr \to sr,i}}{factinputshare_{dr \to sr,i}} PRODSCALAR_{dr \to sr,i}\, multifactorprod2_{dr \to sr,i} \tag{B.96}$$

$$expcommodityd_{sr,c} = BASEEXPORTS_{dr \to sr,c} \left(\frac{PCOMMWORLDEXP_c(t)}{perceivedpexportd_{sr,c}} \right)^{EXPORTP_c} \times WORLDGDPINDX(t)^{GDPPARAM_c} \tag{B.97}$$

$$pexportcommd_{sr,c} = \mathbf{Pexpcomm}_{sr,c} + pexportmargins_{sr,c}\mathbf{Exchangert} \tag{B.98}$$

$$actualpexports_{sr,c} = \frac{actualexports_{sr,c}\, \mathbf{Pexpcomm}_{sr,c}}{expcommoditys_{sr,c}} \tag{B.99}$$

$$excessproduction_{sr,dr,c} = \frac{regcdomcomms_{sr,dr,c}}{regdomcommd_{sr,dr,c}} \tag{B.100}$$

$$regcdomcomms_{sr,dr,c} = \left[(\theta^{commregs}_{sr,c})^{\phi^{regcom}_{sr,c}} \xi^{commregs}_{sr,dr,c} \frac{\mathbf{Pcompdomcomms}_{sr,c}}{\mathbf{Pregdomcomm}_{sr,dr,c}} \right]^{\frac{1}{1-\phi^{regcom}_{sr,c}}} \times domcommoditys_{sr,c} \tag{B.101}$$

$$regdomcommd_{sr,dr,c} = \left[(\gamma^{commregd}_{dr,c})^{\eta^{regcom}_{dr,c}} \delta^{commregd}_{sr,dr,c} \frac{\mathbf{Pcompdomcommd}_{dr,c}}{pregdomcomminclmargin_{sr,dr,c}} \right]^{\frac{1}{1-\eta^{regcom}_{dr,c}}} \times domcomdemand_{dr,c} \tag{B.102}$$

$$importdemand_{dr,c} = \left[(\gamma^{commd}_{dr,c})^{\eta^{com}_{dr,c}} \delta^{commdimp}_{dr,c} \frac{\mathbf{Pperceivedcompcommd}_{dr,c}}{perceivedimportp_{dr,c}} \right]^{\frac{1}{1-\eta^{com}_{dr,c}}} \times totalcomdemand_{dr,c} \tag{B.103}$$

$$\begin{aligned} loandemands_{dr=DReg1,c} &= FISCOMMMAP_c \textstyle\sum_{mf} finintservdemands_{mf,lt} \\ loandemands_{dr=DReg2,c} &= 0 \end{aligned} \tag{B.104}$$

$$totalcomdemand_{dr,c} = \sum_{g}(govtconsump_{g,dr,c}) + hhldconsump_{dr,c} + investconsumpq_{dr,c} + \sum_{i}(indconsump_{dr,i,c}) + marginconsumpq_{dr,c} + totalfencingdemand_{dr,c} + totalplandemand_{dr,c} + loandemands_{dr,c} + OPEXDEMANDBYTIME_{dr}(t) \times WASTEMAP_{c} \quad \text{(B.105)}$$

$$indconsump_{dr,i,c} = intinputcoeff_{dr,i,c} \frac{interinputshare_{dr,i}}{factinputshare_{dr,i}} \times \begin{cases} compfactoru_{dr,i} & \text{for} \quad OPERABILITY_{i} = 1 \quad \text{or} \quad STOCKPILECOMM_{c} = 1 \\ effectcompfactoru_{dr,i} & \text{for} \quad OPERABILITY_{i} < 1 \quad \& \quad STOCKPILECOMM_{c} < 1 \end{cases} \quad \text{(B.106)}$$

$$totalindconsump_{dr,c} = \sum_{i}(indconsump_{dr,i,c}) \quad \text{(B.107)}$$

$$pimpcommnz_{dr,c} = \frac{PCOMMWORLDIMP_{c}(t)}{\textbf{Exchangert}} + pimportmargins_{dr,c} + pimporttariffs_{c} \quad \text{(B.108)}$$

$$actualpccd_{dr,c} = \frac{pimpcommnz_{dr,c}\, importdemand_{dr,c} + \textbf{Pcompdomcommd}_{dr,c}\, domcomdemand_{dr,c}}{qcompcommd_{dr,c}} \quad \text{(B.109)}$$

$$qcompcommd_{dr,c} = \gamma^{commd}_{dr,c} \left[\delta^{commdimp}_{dr,c}(importdemand_{dr,c})^{\eta^{com}_{dr,c}} + \delta^{commddom}_{dr,c}(domcomdemand_{dr,c})^{\eta^{com}_{dr,c}}\right]^{\frac{1}{\eta^{com}_{dr,c}}} \quad \text{(B.110)}$$

$$domcomdemand_{dr,c} = \left[(\gamma^{commd}_{dr,c})^{\eta^{com}_{dr,c}}\, \delta^{commddom}_{dr,c} \frac{\textbf{Pperceivedcompcommd}_{dr,c}}{\textbf{Pcompdomcommd}_{dr,c}}\right]^{\frac{1}{1-\eta^{com}_{dr,c}}} \times totaldemand_{dr,c} \quad \text{(B.111)}$$

$$actualpccs_{sr,c} = \frac{pexpcommnz_{sr,c}\, expcommoditys_{sr,c} + \textbf{Pcompdomcomms}_{sr,c}\, domcommoditys_{sr,c}}{qcompcomms_{sr,c}} \quad \text{(B.112)}$$

$$qcompcomms_{sr,c} = \theta^{commsdexp}_{sr,c} \left[\xi^{commsexp}_{sr,c}\, (expcommoditys_{sr,c})^{\phi^{com}_{sr,c}} + \xi^{commsdom}_{sr,c}\, (domcommoditys_{sr,c})^{\phi^{com}_{sr,c}}\right]^{\frac{1}{\phi^{com}_{sr,c}}} \quad \text{(B.113)}$$

$$actualpcdcd_{dr,c} = \frac{\sum_{sr}(pregdomcomminclmargin_{sr,dr,c}\, regdomcommd_{sr,dr,c})}{qdomcommd_{dr,c}} \quad \text{(B.114)}$$

$$qdomcommd_{dr,c} = \gamma^{commregd}_{dr,c} \left[\sum_{sr}\left(\delta^{commregd}_{sr,dr,c}(regdomcommd_{sr,dr,c})^{\eta^{regcom}_{dr,c}}\right)\right]^{\frac{1}{\eta^{regcom}_{dr,c}}} \quad \text{(B.115)}$$

$$actualpcdcs_{sr,c} = \frac{\sum_{dr}\left(\textbf{Pregdomcomm}_{sr,dr,c}\, regcdomcomms_{sr,dr,c}\right)}{qdomcomms_{sr,c}} \quad \text{(B.116)}$$

$$qdomcomms_{sr,c} = \theta^{commregs}_{sr,c} \left[\sum_{dr} \left(\xi^{commregs}_{sr,dr,c} (regcdomcomms_{sr,dr,c})^{\phi^{regcom}_{sr,c}} \right) \right]^{\frac{1}{\phi^{regcom}_{sr,c}}} \tag{B.117}$$

$$actualpcindustrys_{sr,i} = \frac{\sum_{dr} \left(indcommoditys_{sr,i,c}\, \mathbf{Pcompcomms}_{sr,i} \right)}{qcindustrys_{sr,i}} \tag{B.118}$$

$$qcindustrys_{sr,i} = scalecommsup1_{sr,i} \left[\sum_{c} \left(sharecommsup1_{sr,i,c} (indcommoditys_{sr,i,c})^{\phi^{comsup}_{sr,i}} \right) \right]^{\frac{1}{\phi^{comsup}_{sr,i}}} \tag{B.119}$$

$$actualpfiinputs_{dr,i} = \frac{factinputshare_{dr,i}\, \mathbf{Pcfact}_{dr,i} + interinputshare_{dr,i}\, \mathbf{Pintinputs}_{dr,i}}{qfiinputs_{dr,i}} \tag{B.120}$$

$$\begin{aligned} qfiinputs_{dr,i} = scalefi1_{dr,i} \Big[& sharefi1_{input=InterI,dr,i} \left(interinputshare_{dr,i} \right)^{\eta^{fi}_{dr,i}} \\ & + sharefi1_{input=FactsI,dr,i} \left(factinputshare_{dr,i} \right)^{\eta^{fi}_{dr,i}} \Big]^{\frac{1}{\eta^{fi}_{dr,i}}} \end{aligned} \tag{B.121}$$

$$factinputshare_{dr,i} = \left[(scalefi1_{dr,i})^{\eta^{fi}_{dr,i}} (sharefi1_{input=FactsI,dr,i}) \frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pcfact}_{dr,i}} \right]^{\frac{1}{1-\eta^{fi}_{dr,i}}} \tag{B.122}$$

$$interinputshare_{dr,i} = \left[(scalefi1_{dr,i})^{\eta^{fi}_{dr,i}} (sharefi1_{input=InterI,dr,i}) \frac{\mathbf{Pfiinputs}_{dr,i}}{\mathbf{Pintinputs}_{dr,i}} \right]^{\frac{1}{1-\eta^{fi}_{dr,i}}} \tag{B.123}$$

$$actualpintinputs_{dr,i} = \frac{\sum_{c} \left(intinputcoeff_{dr,i,c}\, \mathbf{Pcompcommd}_{dr,c} \right)}{qintinputs_{dr,i}} \tag{B.124}$$

$$qintinputs_{dr,i} = scalecominput1_{dr,i} \left[\sum_{c} (sharecominput1_{dr,i,c} \left(intinputcoeff_{dr,i,c} \right)^{\eta^{cominput}_{dr,i}}) \right]^{\frac{1}{\eta^{cominput}_{dr,i}}} \tag{B.125}$$

$$intinputcoeff_{dr,i,c} = \left[(scalecominput1_{dr,i})^{\eta^{cominput}_{dr,i}}\; sharecominput1_{dr,i,c} \frac{\mathbf{Pintinputs}_{dr,i}}{\mathbf{Pcompcommd}_{dr,c}} \right]^{\frac{1}{1-\eta^{cominput}_{dr,i}}} \tag{B.126}$$

$$\eta^{com}_{dr,c} = \frac{\epsilon^{comsub}_{dr,c} - 1}{\epsilon^{comsub}_{dr,c}} \tag{B.127}$$

$$\eta^{cominput}_{dr,i} = \frac{\epsilon^{cominput}_{dr,i} - 1}{\epsilon^{cominput}_{dr,i}} \tag{B.128}$$

$$\eta^{fi}_{dr,i} = \frac{\epsilon^{fi}_{dr,i} - 1}{\epsilon^{fi}_{dr,i}} \tag{B.129}$$

$$\eta^{regcom}_{dr,c} = \frac{\epsilon^{regcom}_{dr,c} - 1}{\epsilon^{regcom}_{dr,c}} \tag{B.130}$$

$$\phi^{comsup}_{sr,i} = \frac{\psi^{comsup}_{sr,i} + 1}{\psi^{comsup}_{sr,i}} \tag{B.131}$$

$$\phi^{com}_{sr,c} = \frac{\psi^{com}_{sr,c} + 1}{\psi^{com}_{sr,c}} \tag{B.132}$$

$$\phi^{regcom}_{sr,c} = \frac{\psi^{regcom}_{sr,c} + 1}{\psi^{regcom}_{sr,c}} \tag{B.133}$$

$$pimporttariffs_c = \frac{PCOMMWORLDIMP_c(t)}{\mathbf{Exchangert}} \, ADVALOREMIMPTARIFF_c(t) \tag{B.134}$$

$$\begin{aligned} importtariffs_{g=CentralG,dr} &= \textstyle\sum_c (pimporttariffs_c \, importdemand_{dr,c}) \\ importtariffs_{g=LocalG,dr} &= 0 \end{aligned} \tag{B.135}$$

$$\begin{aligned} actualpercccd_{dr,c} = \max \Bigg[1, \frac{1}{qcompcommd_{dr,c}} \Big(& perceivedimportp_{dr,c} \, importdemand_{dr,c} \\ &+ \mathbf{Pcompdomcommd}_{dr,c} \, domcomdemand_{dr,c}\Big)\Bigg] \end{aligned} \tag{B.136}$$

$$perceivedpexportd_{sr,c} = (1 + ADVALOREMEXPORTP_c(t)) \, pexportcommd_{sr,c} \tag{B.137}$$

$$perceivedimportp_{dr,c} = (1 + ADVALOREMIMPORTP_c(t)) \, \frac{PCOMMWORLDIMP_c(t)}{\mathbf{Exchangert}} \tag{B.138}$$

$$sharecominput1_{dr,i,c} = \begin{cases} fisharecominput_{IOag \to i,c} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{cominput}_{dr,i,c} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.139}$$

$$scalecominput1_{dr,i} = \begin{cases} fisalecominput_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{cominput}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.140}$$

$$sharefi1_{input,dr,i} = \begin{cases} fisharefiinput_{IOag \to i,input} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{fi}_{input,dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.141}$$

$$scalecommsup1_{sr,i} = \begin{cases} fiscalecomsup_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ \theta^{comsup}_{sr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.142}$$

$$sharecommsup1_{sr,i,c} = \begin{cases} fisharecomsup_{IOag \to i,c} & \text{if} \quad i = \text{agricultural industry} \\ \xi^{comsup}_{sr,i,c} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.143}$$

$$scalefi1_{dr,i} = \begin{cases} fiscalefiinput_{IOag \to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{fi}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.144}$$

B.6 Factors module equations

B.6.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Agfarmsystemprod}_{dr=DReg1,i}\right) = \frac{actualagriprod_{dr=DReg1,i} - \mathbf{Agfarmsystemprod}_{dr=DReg1,i}}{TIME\ STEP} \times \left(1 - OVERRIDEMAP_i\right) \tag{B.145}$$

$$\frac{d}{dt}\left(\mathbf{Pcfact}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcfact_{dr,i} - \mathbf{Pcfact}_{dr,i}\right) \tag{B.146}$$

$$\begin{aligned} \frac{d}{dt}\left(\mathbf{Pfact}_{h=CAP,dr,i}\right) &= \frac{1}{\tau_{prices}}\left(actualpcapital_{dr,i} - \mathbf{Pfact}_{h=CAP,dr,i}\right) \\ \frac{d}{dt}\left(\mathbf{Pfact}_{h=LAB,dr,i}\right) &= \left(\left(\frac{1}{labratio_{dr,i}}\right)^{\alpha^{plab}} - 1\right)\mathbf{Pfact}_{h=LAB,dr,i} \end{aligned} \tag{B.147}$$

B.6.2 Auxiliaries

$$actualpcfact_{dr,i} = \frac{\sum_h \left(factorsd_{h,dr,i}\,\mathbf{Pfact}_{h,dr,i}\right)}{qcompfactd_{dr,i}} \tag{B.148}$$

$$qcompfactd_{dr,i} = factscalep1_{dr,i}\left[\sum_h \left(factsharep1_{h,dr,i}\left(factorsd_{h,dr,i}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}} \tag{B.149}$$

$$\eta^{fact}_{dr,i} = \frac{\epsilon^{fact}_{dr,i} - 1}{\epsilon^{fact}_{dr,i}} \tag{B.150}$$

$$actualpcapital_{dr,i} = \frac{capitaltyped_{cap=BuilC,dr,i}\,\mathbf{Pbuiltcap}_{dr,i}}{qcapitald_{dr,i}} + \frac{capitaltyped_{cap=NatC,dr,i}\,\mathbf{Pcompnaturalcapd}_{dr,i}}{qcapitald_{dr,i}} \tag{B.151}$$

$$qcapitald_{dr,i} = scalecc1_{dr,i}\left[\sum_{cap}\left(sharecc1_{cap,dr,i}(capitaltyped_{cap,dr,i})^{\eta^{cc}_{dr,i}}\right)\right]^{\frac{1}{\eta^{cc}_{dr,i}}} \tag{B.152}$$

$$labratio_{dr,i} = \frac{\sum_i (factorss_{h=LAB,dr,i})}{\sum_i (factorsd_{h=LAB,dr,i})} \tag{B.153}$$

$$\begin{aligned} factorss_{h=LAB,dr,i} &= adjustedindlaboursup_{dr,i} \\ factorss_{h=CAP,dr,i} &= ccapitals_{dr,i} \end{aligned} \tag{B.154}$$

$$factorsd_{h,dr,i} = \left[(factscalep1_{dr,i})^{\eta^{fact}_{dr,i}}\ factsharep1_{h,dr,i}\ \frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} compfactord_{dr,i} \times (1 - RWFACTRT_{h,dr}) \tag{B.155}$$

$$factorsu_{h,dr,i} = \frac{\min[factorsd_{h,dr,i}, factorss_{h,dr,i}]}{1 - RWFACTRT_{h,dr}} \tag{B.156}$$

$$compfactoru_{dr,i} = factscalep1_{dr,i}\left[\sum_h \left(factsharep1_{h,dr,i}\left(\frac{factorsu_{h,dr,i}}{1 - RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}} \tag{B.157}$$

$$effectfactorsu_{h,dr,i} = \frac{\min[effectfactorsd_{h,dr,i}, effectfactorss_{h,dr,i}]}{1 - RWFACTRT_{h,dr}} \tag{B.158}$$

$$effectfactorsd_{h,dr,i} = \left[(factscalep1_{dr,i})^{\eta^{fact}_{dr,i}}\ factsharep1_{h,dr,i}\ \frac{\mathbf{Pcfact}_{dr,i}}{\mathbf{Pfact}_{h,dr,i}}\right]^{\frac{1}{1-\eta^{fact}_{dr,i}}} effectcompfactord_{dr,i} \times (1 - RWFACTRT_{h,dr}) \tag{B.159}$$

$$effectfactorss_{h,dr,i} = factorss_{h,dr,i} \times OPERABILITY_{sr\to dr,i} \tag{B.160}$$

$$effectcompfactoru_{dr,i} = factscalep1_{dr,i}\left[\sum_h \left(factsharep1_{h,dr,i}\left(\frac{effectfactorsu_{h,dr,i}}{1 - RWFACTRT_{h,dr}}\right)^{\eta^{fact}_{dr,i}}\right)\right]^{\frac{1}{\eta^{fact}_{dr,i}}} \tag{B.161}$$

$$factsharep1_{h,dr,i} = \begin{cases} fisharefactorinput_{IOag\to i,h} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{fact}_{h,dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.162}$$

$$factscalep1_{dr,i} = \begin{cases} fiscalefactorinput_{IOag\to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{fact}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.163}$$

$$actualagriprod_{dr,i} = \begin{cases} agriprodrt_{IOag\to i} & \text{if} \quad i = \text{agricultural industry} \\ 0 & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.164}$$

$$multifactorprod2_{dr,i} = \begin{cases} \mathbf{Agfarmsystemprod}_{dr=DReg1,i} & \text{if} \quad i = \text{agricultural industry} \\ mfpadjusted_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.165}$$

$$mfpadjusted_{dr,i} = MULTIFACTORPROD_{dr,i}\ \left(1 + ADJUSTRATE)^{Time} - 1\right) \tag{B.166}$$

B.7 Labour module equations

B.7.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Indlaboursup}_{dr,i}\right) = newlabsupply_{dr,i} + reallocatedlab_{dr,i} - labtoreallocate_{dr,i} \tag{B.167}$$

B.7.2 Auxiliaries

$$reallocatedlab_{dr,i} = \sum_i labtoreallocate_{dr,i} \times \frac{factorsd_{h=LAB,dr,i} \times \frac{1}{LABINDEX_{dr,i}(t)}}{\sum_i \left(factorsd_{h=LAB,dr,i} \times \frac{1}{LABINDEX_{dr,i}(t)}\right)} \tag{B.168}$$

$$labtoreallocate_{dr,i} = \mathbf{Indlaboursup}_{dr,i}\, MAXREALLOCATERT \tag{B.169}$$

$$newlabsupply_{dr,i} = netincreaselab_{dr} \frac{\mathbf{Indlaboursup}_{dr,i}}{\sum_i \mathbf{Indlaboursup}_{dr,i}} \tag{B.170}$$

$$netincreaselab_{dr} = \frac{1}{\tau}\left(\sum_{sr} reglaboursupply_{sr,dr} - \sum_i \mathbf{Indlaboursup}_{dr,i}\right) \tag{B.171}$$

$$reglaboursupply_{sr,dr} = reglabourest_{sr,dr}\, LSFCONVERT_{dr} \tag{B.172}$$

$$reglabourest_{sr,dr} = \begin{cases} mecforce_{sr} - MECTRANSOUT_{sr} & \text{for} \quad dr = sr \\ MECTRANSOUT_{sr} & \text{for} \quad dr \neq sr \end{cases} \tag{B.173}$$

$$mecforce_{sr} = (labourforce_{sr} - unavailablelab_{sr})\, MECRATIO_{sr} \tag{B.174}$$

$$labourforce_{sr} = (1 - LABFORCEADJUST_{sr})\, WORKINGAGEPOP_{sr}(t)\, PARTICIPATIONRT_{sr}(t) \tag{B.175}$$

$$unavailablelab_{sr} = (1 - LABFORCEADJUST_{sr})\, WORKINGAGEPOP_{sr}(t)\, FUNEMPLOYRT \tag{B.176}$$

$$adjustedindlaboursup_{dr,i} = \mathbf{Indlaboursup}_{dr,i}\, LABINDEX_{dr,i}(t) \tag{B.177}$$

B.8 Capital module equations

B.8.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Builtcapital}_{dr,i}\right) = netcapitalchange_{dr,i} + newcapital_{dr,i} - depreciation_{dr,i} \tag{B.178}$$

$$\frac{d}{dt}(\mathbf{Naturalcapital}_{dr,nct}) = CONVERSIONRT_{dr,nct}\, \mathbf{Naturalcapital}_{dr,nct} \tag{B.179}$$

$$\frac{d}{dt}\left(\mathbf{Pbuiltcap}_{dr,i}\right) = \left(\left(\frac{1}{builtratio_{dr,i}}\right)^{\alpha^{pbuiltcap}} - 1\right)\mathbf{Pbuiltcap}_{dr,i} \tag{B.180}$$

$$\frac{d}{dt}\left(\mathbf{Pnaturalcap}_{dr,i,nct}\right) = \left(\left(\frac{1}{naturalcapratio_{dr,i,nct}}\right)^{\alpha^{pnatcap}} - 1\right)\mathbf{Pnaturalcap}_{dr,i,nct} \tag{B.181}$$

$$\frac{d}{dt}\left(\mathbf{Pcompnaturalcapd}_{dr,i}\right) = \frac{1}{\tau_{prices}}\left(actualpcnaturalcapd_{dr,i} - \mathbf{Pcompnaturalcapd}_{dr,i}\right) \tag{B.182}$$

$$\frac{d}{dt}\left(\mathbf{Pcompnaturalcaps}_{dr,nct}\right) = \frac{1}{\tau_{prices}}\left(actualpcnaturalcaps_{dr,nct} - \mathbf{Pcompnaturalcaps}_{dr,nct}\right) \tag{B.183}$$

$$\frac{d}{dt}\left(\mathbf{Rcapincome}_{dr}\right) = \frac{1}{\tau_{income}}\left(capitalincome_{dr} - \mathbf{Rcapincome}_{dr}\right) \tag{B.184}$$

B.8.2 Auxiliaries

$$builtratio_{dr,i} = \frac{builts_{dr,i}}{capitaltyped_{cap=BuilC,dr,i}} \tag{B.185}$$

$$builts_{dr,i} = \mathbf{Builtcapital}_{dr,i}\, KSFCONVERT_{dr,i}\, CAPINDEX_{dr,i}(t) \tag{B.186}$$

$$ccapitals_{dr,i} = \left[sharecc1_{cap=BuilC,dr,i}\left(builts_{dr,i}\right)^{\eta^{cc}_{dr,i}} + sharecc1_{cap=NatC,dr,i}\left(naturalcapitalsq_{dr,i}\right)^{\eta^{cc}_{dr,i}}\right]^{\frac{1}{\eta^{cc}_{dr,i}}} \times scalecc1_{dr,i} \tag{B.187}$$

$$naturalcapitalsq_{dr,i} = compnaturalcaps_{dr,i}\, NATCAPCONVERT_{dr,i} \tag{B.188}$$

$$compnaturalcaps_{dr,i} = \gamma^{natcap}_{dr,i}\left[\sum_{nct}\left(\delta^{natcap}_{dr,i,nct}\left(indnaturalcaps_{dr,i,nct}\right)^{\eta^{natcap}_{dr,i}}\right)\right]^{\frac{1}{\eta^{natcap}_{dr,i}}} \tag{B.189}$$

$$capitalincome_{dr} = \frac{\sum_i\left(factorsu_{h=CAP,dr,i}\,\mathbf{Pfact}_{h=CAP,dr,i}\right)}{1 - RWFACTRT_{h=CAP,dr}} + capregtransout_{DReg1\leftrightarrow DReg2} + \sum_i\left(\mathbf{Industrybalance}_{dr,i}\right) - bussloanpayments_{dr} - \sum_c vmunopex_{dr,c,mf=Buss} \tag{B.190}$$

$$capregtransout_{dr} = \mathbf{Rcapincome}_{dr} \times CREGTRANSRT_{dr} \tag{B.191}$$

$$capentertrans_{dr} = \mathbf{Rcapincome}_{dr} \times CENTTRANSRT_{dr} \tag{B.192}$$

$$capgovttrans_{g,dr} = \mathbf{Rcapincome}_{dr} \times CGOVTTRANSRT_{g,dr} \tag{B.193}$$

$$caplocalhhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CHHLDTRANSRT_{dr} \tag{B.194}$$

$$capreghhldtrans_{dr} = \mathbf{Rcapincome}_{dr} \times CRHTRANSRT_{dr} \tag{B.195}$$

$$newcapital_{dr,i} = mobileinvest_{dr,i} + immobileinvest_{dr,i} \tag{B.196}$$

$$mobileinvest_{dr,i} = \left[\frac{aggregateinvestv1_{dr}\left(1 - NONPRODINVESTSH(t)\right)}{\mathbf{Pinvestcc}_{dr}} + \sum_c\left(SETINVESTCQ_{dr,c}\right)\right] \times MOBILESH_{dr,i}\, mobileinvestsh_{dr,i} \tag{B.197}$$

$$immobileinvest_{dr,i} = \left[\frac{aggregateinvestv1_{dr}\,(1 - NONPRODINVESTSH(t))}{\mathbf{Pinvestcc}_{dr}} + \sum_c (SETINVESTCQ_{dr,c}) \right] \times (1 - MOBILESH_{dr,i})\, capincomesh_{dr,i} \qquad (B.198)$$

$$mobileinvestsh_{dr,i} = \frac{mobileinvest1_{dr,i}}{\sum_i (mobileinvest1_{dr,i})} ALLOCATESH_{dr} + INVESTCONSTSH_{dr,i} \qquad (B.199)$$

$$mobileinvest1_{dr,i} = (INVESTPARAM_{dr,i}\, netreturn_{dr,i})^{EINVEST_{dr,i}} \times capincomesh_{dr,i} \qquad (B.200)$$

$$netreturn_{dr,i} = grossreturn_{dr,i} - RDEP_{dr,i} \qquad (B.201)$$

$$grossreturn_{dr,i} = \frac{\mathbf{Pbuiltcap}_{dr,i}\, KSFCONVERT_{dr,i}}{\mathbf{Pinvestcc}_{dr}} \qquad (B.202)$$

$$capincomesh_{dr,i} = \frac{indcapincome_{dr,i}}{\sum_i (indcapincome_{dr,i})} \qquad (B.203)$$

$$indcapincome_{dr,i} = builtuse_{dr,i}\, \mathbf{Pbuiltcap}_{dr,i} \qquad (B.204)$$

$$builtuse_{dr,i} = \min \left[\mathbf{Builtcapital}_{dr,i}\, KSFCONVERT_{dr,i}\,,\, capitaltyped_{cap=BuilC,dr,i} \right] \qquad (B.205)$$

$$depreciation_{dr,i} = \mathbf{Builtcapital}_{dr,i} \left[RDEP_{dr,i}\,(1 + DEPSHFT) \right] \qquad (B.206)$$

$$naturalcapd_{dr,i,nct} = \left[(\gamma^{natcap}_{dr,i})^{\eta^{natcap}_{dr,i}}\, \delta^{natcap}_{dr,i,nct} \frac{\mathbf{Pcompnaturalcapd}_{dr,i}}{\mathbf{Pnaturalcap}_{dr,i,nct}} \right]^{\frac{1}{1-\eta^{natcap}_{dr,i}}} \frac{capitaltyped_{cap=NatC,dr,i}}{NATCAPCONVERT_{dr,i}} \qquad (B.207)$$

$$\begin{aligned} capitaltyped_{cap=BuilC,dr,i} &= \left[(scalecc1_{dr,i})^{\eta^{cc}_{dr,i}}\, sharecc1_{cap=BuilC,dr,i} \frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pbuiltcap}_{dr,i}} \right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \\ &\quad \times factorsd_{h=CAP,dr,i} \\ capitaltyped_{cap=NatC,dr,i} &= \left[(scalecc1_{dr,i})^{\eta^{cc}_{dr,i}}\, sharecc1_{cap=NatC,dr,i} \frac{\mathbf{Pfact}_{h=CAP,dr,i}}{\mathbf{Pcompnaturalcapd}_{cap=NatC,dr,i}} \right]^{\frac{1}{1-\eta^{cc}_{dr,i}}} \\ &\quad \times factorsd_{h=CAP,dr,i} \end{aligned} \qquad (B.208)$$

$$\eta^{cc}_{dr,i} = \frac{\epsilon^{cc}_{dr,i} - 1}{\epsilon^{cc}_{dr,i}} \qquad (B.209)$$

$$naturalcapratio_{dr,i,nct} = \frac{indnaturalcaps_{dr,i,nct}}{naturalcaptyped_{dr,i,nct}} \qquad (B.210)$$

$$indnaturalcaps_{dr,i,nct} = \frac{indnaturalcaps1_{dr,i,nct}}{\sum_i (indnaturalcaps1_{dr,i,nct})} \mathbf{Naturalcapital}_{dr,nct} + agrilandsup_{dr,i,nct} \qquad (B.211)$$

$$actualpcnaturalcaps_{dr,nct} = \frac{\sum_i \left(indnaturalcaps1_{dr,i,nct}\,\mathbf{Pnaturalcap}_{dr,i,nct}\right)}{\mathbf{Naturalcapital}_{dr,nct}} \tag{B.212}$$

$$indnaturalcaps1_{dr,i,nct} = \left[(\theta^{natcap}_{dr,nct})^{\phi^{natcap}_{dr,nct}}\,\xi^{natcap}_{dr,i,nct}\,\frac{\mathbf{Pcompnaturalcaps}_{dr,nct}}{\mathbf{Pnaturalcap}_{dr,i,nct}}\right]^{\frac{1}{1-\phi^{natcap}_{dr,nct}}}\,\mathbf{Naturalcapital}_{dr,nct} \tag{B.213}$$

$$\phi^{natcap}_{dr,nct} = \frac{\psi^{natcap}_{dr,nct}+1}{\psi^{natcap}_{dr,nct}} \tag{B.214}$$

$$actualpcnaturalcapd_{dr,i} = \frac{\sum_{nct}\left(naturalcapd_{dr,i,nct}\,\mathbf{Pnaturalcap}_{dr,i,nct}\right)}{actualindcompnaturalcaps_{dr,i}} \tag{B.215}$$

$$actualindcompnaturalcaps_{dr,i} = \gamma^{nattyp}_{dr,i}\left[\sum_{nct}\left(\delta^{natcap}_{dr,i,nct}\,(naturalcapd_{dr,i,nct})^{\eta^{natcap}_{dr,i}}\right)\right]^{\frac{1}{\eta^{natcap}_{dr,i}}} \tag{B.216}$$

$$\eta^{natcap}_{dr,i} = \frac{\epsilon^{natcap}_{dr,i}-1}{\epsilon^{natcap}_{dr,i}} \tag{B.217}$$

$$\begin{aligned} vmunopex_{dr,c,mf} = & OPEXDEMANDBYTIME_{dr}(t) \times OPEXRESPBYTIME_{mf}(t) \times \mathbf{Pcompcommd}_{dr,c} \\ & \times WASTEMAP_c \end{aligned} \tag{B.218}$$

$$\begin{aligned} bussloanpayments_{dr=DReg1} &= \textstyle\sum_{lt} totalloanpayments_{rt=Normal,mf=Buss,lt} \\ bussloanpayments_{dr=DReg2} &= 0 \end{aligned} \tag{B.219}$$

$$sharecc1_{cap,dr,i} = \begin{cases} fisharecapitalinput_{IOag\to i,cap} & \text{if} \quad i = \text{agricultural industry} \\ \delta^{cc}_{cap,dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.220}$$

$$scalecc1_{dr,i} = \begin{cases} fiscalecapitalinput_{IOag\to i} & \text{if} \quad i = \text{agricultural industry} \\ \gamma^{cc}_{dr,i} & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.221}$$

$$netcapitalchange_{dr,i} = \begin{cases} netratelandusechange_{IOag\to i}\,\mathbf{Builtcapital}_{dr,i} & \text{if} \quad i = \text{agricultural industry} \\ 0 & \text{if} \quad i = \text{non-agricultural industry} \end{cases} \tag{B.222}$$

$$agrilandsup_{dr,i,nct} = \begin{cases} industryland_{IOag\to i} & \text{if} \quad nct = \text{Land1} \\ 0 & \text{if} \quad nct \neq \text{Land1} \end{cases} \tag{B.223}$$

B.9 Investment and savings module equations

B.9.1 Stocks

$$\frac{d}{dt}(\mathbf{Casurplus}) = \frac{1}{\tau_{casurplus}}(actualcasurplus - \mathbf{Casurplus}) \tag{B.224}$$

$$\frac{d}{dt}(\mathbf{Interestrt}) = \begin{cases} \frac{1}{\tau}(ACTUALINTERESTRT(t) - \mathbf{Interestrt}) & \text{for} \quad t < 9 \\ \frac{1}{\tau_{interest}}(taylorinterestrt - \mathbf{Interestrt}) & \text{for} \quad t \geq 9 \end{cases} \tag{B.225}$$

$$\frac{d}{dt}(\mathbf{Pinvestcc}_{dr}) = \frac{1}{\tau_{prices}}(actualpinvestcc_{dr} - \mathbf{Pinvestcc}_{dr}) \tag{B.226}$$

B.9.2 Auxiliaries

$$actualcasurplus = rwexpenditure - rwincome \tag{B.227}$$

$$\begin{aligned} desiredinterestrt = &\mathbf{Inflationrt} + INTERESTCONST + INTERESTINFLW\,(\mathbf{Inflationrt} - 0.02) \\ &- INTERESTGDPW\; gdpgap \end{aligned} \tag{B.228}$$

$$gdpgap = \frac{realgdp}{NATURALGDP(t)} - 1 \tag{B.229}$$

$$realinterestrt = \mathbf{Interestrt} - \mathbf{Inflationrt} \tag{B.230}$$

$$actualpinvestcc_{dr} = \frac{\sum_c \left(disinvestconsump_{dr,c}\, \mathbf{Pcompcommd}_{dr,c}\right)}{qinvestcc_{dr}} \tag{B.231}$$

$$qinvestcc_{dr} = \gamma_{dr}^{investc} \left[\sum_c \left(\delta_{dr,c}^{investc}\,(disinvestconsump_{dr,c})^{\eta_{dr}^{investc}}\right)\right]^{\frac{1}{\eta_{dr}^{investc}}} \tag{B.232}$$

$$\eta_{dr}^{investc} = \frac{\epsilon_{dr}^{investc} - 1}{\epsilon_{dr}^{investc}} \tag{B.233}$$

$$disinvestconsump_{dr,c} = \left[(\gamma_{dr}^{investc})^{\eta_{dr}^{investc}}\,\delta_{dr,c}^{investc}\,\frac{\mathbf{Pinvestcc}_{dr}}{\mathbf{Pcompcommd}_{dr,c}}\right]^{\frac{1}{1-\eta_{dr}^{investc}}} \frac{aggregateinvestv1_{dr}}{\mathbf{Pinvestcc}_{dr}} \tag{B.234}$$

$$\begin{aligned} aggregateinvestv_{dr} = &\Bigg[\Bigg(realinterestrt\; ALPHA + \sum_{dr}(savingstotal_{dr}\; BETA) \\ &+ INVESTCONST\Bigg)\frac{\mathbf{Holdregvaladd}_{dr}}{\sum_{dr}\mathbf{Holdregvaladd}_{dr}} + REGINVESTCONST_{dr}\Bigg] \\ &\times (1 - INVESTINDIRECTTAXRT_{dr}) \end{aligned} \tag{B.235}$$

$$aggregateinvestv1_{dr} = aggregateinvestv_{dr} - (investadjustcap + investadjustland)\,\frac{aggregateinvestv_{dr}}{\sum_{dr} aggregateinvestv_{dr}} \tag{B.236}$$

$$savingstotal_{dr} = rwsavings_{dr} + regsavings_{dr} - savregtransout_{dr} \tag{B.237}$$

$$savingstotal1_{dr} = savingstotal_{dr} + \sum_{mf,lt} (totalloanpayments_{rt=FinIntServ,mf,lt})\,\frac{savingstotal_{dr}}{\sum_{dr} savingstotal_{dr}} \tag{B.238}$$

$$rwsavings_{dr} = rwsavingstotal\,\frac{\mathbf{Holdregvaladd}_{dr}}{\sum_{dr} \mathbf{Holdregvaladd}_{dr}} + RWREGSAVCONST_{dr} \tag{B.239}$$

$$rwsavingstotal = \begin{cases} ACRWSAVINGS & \text{if} \quad Time < 4 \\ 100\,\mathbf{Interestrt} \times NZINTERESTWEIGHT & \\ +WORLDGDPINDX(t)\,GDPWEIGHT & \\ +RWSAVCONST + NETTRANSFERS(t) & \text{if} \quad Time \geq 4 \end{cases} \tag{B.240}$$

$$regsavings_{dr} = entsavtrans_{dr} + hhldsavings_{dr} + \sum_{g} (govtsavings_{g,dr}) + savregtransout_{DReg1 \leftrightarrow DReg2} \tag{B.241}$$

$$savregtransout_{dr} = SAVREGTRANSBS_{dr}\,\frac{\mathbf{Rhhldincome}_{dr}}{BASEHHLDACCOUNT_{dr}} \tag{B.242}$$

$$investconsumpq_{dr,c} = disinvestconsump_{dr,c} + SETINVESTCQ_{dr,c} + muninvestconsump_{dr,c} \tag{B.243}$$

$$\begin{aligned} investindirecttax_{dr} =& aggregateinvestv1_{dr}\,\frac{INVESTINDIRECTTAXRT_{dr}}{1 - INVESTINDIRECTTAXRT_{dr}} \\ &+ MUNCAPINVESTBYTIME_{dr}(t) \times MUNINDTAXRT\,\frac{\mathbf{Gdpindex}}{1000} \end{aligned} \tag{B.244}$$

$$taylorinterestrt = \begin{cases} INTERESTCONST + INTERESTINFLW\,(desiredinflationrt - 0.02) & \\ \quad +INTERESTGDPW\,ACTUALGDPGAP(t) & \text{for} \quad t \leq 3 \\ INTERESTCONSTGFC + INTERESTINFLW\,(\mathbf{Inflationrt} - 0.02) & \\ \quad +INTERESTGDPW\,gdpgap & \text{for} \quad t > 3 \end{cases} \tag{B.245}$$

B.10 Municipal module equations

B.10.1 Stocks

$$\frac{d}{dt}(\mathbf{Loantopay}_{yrmt,rt,mf,lt}) = totaltopay_{yrmt,rt,mf,lt} - loanpayments_{yrmt,rt,mt,lt} \tag{B.246}$$

B.10.2 Auxiliaries

$$finintservdemands_{mf,lt} = totalloanpayments_{rt=Normal,mf,lt} - totalloanpayments_{rt=FinIntServ,mf,lt} \tag{B.247}$$

$$totalloanpayments_{rt,mf,lt} = \sum_{yrmt} loanpayments_{yrmt,rt,mf,lt} \tag{B.248}$$

$$loanpayments_{yrmt,rt,mf,lt} = \begin{cases} maxloanpayment_{yrmt,rt,mf,lt} & \text{if} \quad \mathbf{Loantopay}_{yrmt,rt,mf,lt} > 0 \\ 0 & \text{if} \quad \mathbf{Loantopay}_{yrmt,rt,mf,lt} \leq 0 \end{cases} \tag{B.249}$$

$$totaltopay_{yrmt,rt,mf,lt} = loanpaymentsbyyearloan_{yrmt,rt,mf,lt}\, YEARSFORLOAN \tag{B.250}$$

$$loanpaymentsbyyearloan_{yrmt,rt,mf,lt} = \begin{cases} annualloanpayments_{rt,lt}\, CAPINVESTRESP_{yrmt,mf,lt} \\ \qquad \text{if} \quad LOANYR - 0.5 < Time \leq LOANYR + 0.5 \\ 0 \qquad \text{if} \quad LOANYR - 0.5 \geq Time > LOANYR + 0.5 \end{cases} \tag{B.251}$$

$$annualloanpayments_{rt,lt} = periodicloanpayments_{rt,lt}\, PAYMENTSPERYEAR \tag{B.252}$$

$$\begin{aligned} periodicloanpayments_{rt,lt=Capital} &= investadjustcap \frac{periodicrate_{rt}\,(1+periodicrate_{rt})^{numberpayments}}{((1+periodicrate_{rt})^{numberpayments})-1} \\ periodicloanpayments_{rt,lt=Land} &= investadjustland \frac{periodicrate_{rt}\,(1+periodicrate_{rt})^{numberpayments}}{((1+periodicrate_{rt})^{numberpayments})-1} \end{aligned} \tag{B.253}$$

$$\begin{aligned} investadjustcap = &\sum_{dr,c} \left(muninvestconsump_{dr,c}\, \mathbf{Pcompcommd}_{dr,c}\right) + \sum_{dr} \Bigg(MUNCAPINVESTBYTIME_{dr}(t) \\ &\times MUNINDTAXRT \frac{\mathbf{Gdpindex}}{1000}\Bigg) \end{aligned} \tag{B.254}$$

$$muninvestconsump_{dr,c} = MUNCAPINVESTBYTIME_{dr}(t) \times MUNINVESTRATIO_{c} \tag{B.255}$$

$$investadjustland = \sum_{dr} \left(MUNLANDINVESTBYTIME_{dr}(t) \frac{\mathbf{Gdpindex}}{1000}\right) \tag{B.256}$$

$$numberpayments = YEARSFORLOAN \times PAYMENTSPERYEAR \tag{B.257}$$

$$\begin{aligned} periodicrate_{rt=Normal} &= \frac{LOANRATE}{PAYMENTSPERYEAR} \\ periodicrate_{rt=FinIntServ} &= \frac{LOANRATE-FISSHARE}{PAYMENTSPERYEAR} \end{aligned} \tag{B.258}$$

$$maxloanpayment_{yrmt,rt,mf,lt}(t+1) = \begin{cases} loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t+1) \\ \qquad \text{if} \quad loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t+1) \\ \qquad\qquad \geq loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t) \\ loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t) \\ \qquad \text{if} \quad loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t+1) \\ \qquad\qquad < loanpaymentsbyyearloan_{yrmt,rt,mf,lt}(t) \end{cases} \tag{B.259}$$

B.11 Primary module equations

B.11.1 Stocks

$$\frac{d}{dt}(\mathbf{Shareretirenp}_{ez,agi}) = \frac{NPRETIRE_{ez,agi}}{LRIMPEND_{ez,agi} - LRIMPSTART_{ez,agi}} \tag{B.260}$$

$$\frac{d}{dt}(\mathbf{Shareretirefr}_{ez,agi}) = \frac{FRRETIREez, agi}{LRIMPEND_{ez,agi} - LRIMPSTART_{ez,agi}} \tag{B.261}$$

$$\frac{d}{dt}(\mathbf{Percfsreturnsperha}_{ez,ft,mt}) = \frac{fsreturnsperha_{ez,ft,mt} - \mathbf{Percfsreturnsperha}_{ez,ft,mt}}{ADJUSTTIME} \tag{B.262}$$

$$\begin{aligned}\frac{d}{dt}(\mathbf{Landuse}_{ez,ft,mt}) =& \, actuallanduse_{ez,ft,mt} + landusechangein_{ez,ft,mt} - landusechangeout_{ez,ft,mt} \\ & - farmretirement_{ez,ft,mt} + mitigationin_{ez,ft,mt} - mitigationout_{ez,ft,mt}\end{aligned} \tag{B.263}$$

B.11.2 Auxiliaries

$$actuallanduse_{ez,ft,mt} = \begin{cases} \Big[\sum_{agi} \left(ACTUALLAND_{ez,agi}(t)\, fsshareallocation_{ez,ft,mt,agi}\right) & \\ \quad - \mathbf{Landuse}_{ez,ft,mt}\Big] \frac{1}{TIME\ STEP} & \text{if} \quad Time < KNOWLANDTIME \\ 0 & \text{if} \quad Time \geq KNOWLANDTIME \end{cases} \tag{B.264}$$

$$\begin{aligned}totalfilabour_{IOag} =& \sum_{ez,ft,mt} \left(managsystlabcoefha_{ez,ft,mt}\, \mathbf{Landuse}_{ez,ft,mt}\, MAPTOIOAG_{IOag}\right) \\ & + RESIDUALLABOUR_{IOag}\end{aligned} \tag{B.265}$$

$$mangsystindtaxcoefha_{ez,ft,mt} = \sum_{mofa} \left(MAPTOMANGSYST_{ez,ft,mofa}\, MODFARMINDTAXCOEFHA_{mt,mofa}\right) \tag{B.266}$$

$$\begin{aligned}managsystlabcoefha_{ez,ft,mt} =& \, MAPTMITFF_{mt} \sum_{mofa} \Big[MAPTOMANGSYST_{ez,ft,mofa} \\ & \times \left(MODFARMLABOURCOEFHA_{mt,mofa} + MODFARMLABOURCOEFHA_{mt=Miti04,mofa}\right)\Big]\end{aligned} \tag{B.267}$$

$$\begin{aligned}mangsystdepreccoefha_{ez,ft,mt} =& \, MAPTMITFF_{mt} \sum_{mofa} \Big[MAPTOMANGSYST_{ez,ft,mofa} \\ & \times \left(MODFARMDEPCOEFHA_{mt,mofa} + MODFARMDEPCOEFHA_{mt=Miti04,mofa}\right)\Big]\end{aligned} \tag{B.268}$$

$$modfarmoscoefha_{ez,ft,mt} = \sum_{c} (managsystoutputcoefha_{ez,ft,mt,c} - mangsystinputcoefha_{ez,ft,mt,c}) - managsystlabcoefha_{ez,ft,mt} - mangsystindtaxcoefha_{ez,ft,mt} - mangsystdepreccoefha_{ez,ft,mt} \quad (B.269)$$

$$builtcaprevsh_{ft} = 1 - LNDREVENUESH_{ft} \quad (B.270)$$

$$totalfibuiltcapital_{IOag} = \sum_{ez,ft,mt} \Big[\mathbf{Landuse}_{ez,ft,mt}\, MAPTOIOAG_{IOag,ft} \times \Big(modfarmoscoefha_{ez,ft,mt}\, builtcaprevsh_{ft} + mangsystdepreccoefha_{ez,ft,mt} \Big) \Big] + RESIDUALBCAPITAL_{IOag} \quad (B.271)$$

$$totalfiland_{IOag} = \sum_{ez,ft,mt} \Big(modfarmoscoefha_{ez,ft,mt}\, \mathbf{Landuse}_{ez,ft,mt}\, MAPTOIOAG_{IOag,ft} \times LNDREVENUESH_{ft} \Big) + RESIDUALLAND_{IOag} \quad (B.272)$$

$$\begin{aligned} fifactorinput_{h=CAP,IOag} &= totalfibuiltcapital_{IOag} + totalfiland_{IOag} \\ fifactorinput_{h=LAB,IOag} &= totalfilabour_{IOag} \end{aligned} \quad (B.273)$$

$$\begin{aligned} ficapitalinput_{cap=BuilC,IOag} &= totalfibuiltcapital_{IOag} \\ ficapitalinput_{cap=LandC,IOag} &= totalfiland_{IOag} \end{aligned} \quad (B.274)$$

$$\begin{aligned} fifiinputcoef_{input=InterI,IOag} &= \frac{\sum_c totalfiintinput_{IOag,c}}{\sum_c totalfiintinput_{IOag,c} + totalfilabour_{IOag} + totalfibuiltcapital_{IOag} + totalfiland_{IOag}} \\ fifiinputcoef_{input=FactsI,IOag} &= \frac{totalfilabour_{IOag} + totalfibuiltcapital_{IOag} + totalfiland_{IOag}}{\sum_c totalfiintinput_{IOag,c} + totalfilabour_{IOag} + totalfibuiltcapital_{IOag} + totalfiland_{IOag}} \end{aligned} \quad (B.275)$$

$$totalfiintinput_{IOag,c} = \sum_{ez,ft,mt} (mangsystinputcoef_{ez,ft,mt,c}\, \mathbf{Landuse}_{ez,ft,mt}\, MAPTOIOAG_{IOag,ff}) + RESIDUALINPUT_{IOag,c} \quad (B.276)$$

$$fiintinputcoef_{IOag,c} = \frac{totalfiintinput_{IOag,c}}{\sum_c totalfiintinput_{IOag,c}} \quad (B.277)$$

$$landintensityscalar_{ez,agi} = 1 - \mathbf{Shareretirefr}_{ez,agi} - \mathbf{Shareretirenp}_{ez,agi} \quad (B.278)$$

$$landintensityscalar2_{ez,agi} = \sum_{agi} (landintensityscalar_{ez,agi}\, MAPTOAGIND_{ft,agi}) \quad (B.279)$$

$$forestryintensityscalar_{ez,ft} = \sum_{agi} (\mathbf{Shareretirefr}_{ez,agi}\, MAPTOAGIND_{ft,agi}) \quad (B.280)$$

$$fenceprice = \sum_{c} \left(\mathbf{Pcompcommd}_{dr=DReg1,c} \, FENCEMAP_{c} \right) \tag{B.281}$$

$$councilfeeprice = \sum_{c} \left(\mathbf{Pcompcommd}_{dr=DReg1,c} \, PLANMAP1_{c} \right) \tag{B.282}$$

$$otherfeeprice = \sum_{c} \left(\mathbf{Pcompcommd}_{dr=DReg1,c} \, PLANMAP2_{c} \right) \tag{B.283}$$

$$\begin{aligned} ffmitinputcoefha_{ez,ft,c} = & \sum_{mofa} (modfarmitffinputcoefha_{mofa,c} \, MAPTOMANGSYST_{ez,ft,mofa}) \\ & \times INTENSITYSCALE_{ez,ft} + (1 - INTENSITYSCALE_{ez,ft}) \end{aligned} \tag{B.284}$$

$$\begin{aligned} mangsystinputcoefha_{ez,ft,mt,c} = & \Bigg[ffmitinputcoefha_{ez,ft,c} \, MAPTMITFF_{mt} \\ & + \sum_{mofa} \left(MAPTOMANGSYST_{ez,ft,mofa} \, MODFARMOUTPUTCOEFHA_{mt,mofa,c} \right) \Bigg] \\ & \times landintensityscalar2_{ez,ft} + FARMFRINPUTCOEFHA_{c} \, forestryintensityscalar_{ez,ft} \end{aligned} \tag{B.285}$$

$$\begin{aligned} ffmitoutputcoefha_{ez,ft,c} = & \sum_{mofa} (modfarmmitffoutputcoefha_{mofa,c} \, MAPTOMANGSYST_{ez,ft,mofa}) \\ & \times INTENSITYSCALE_{ez,ft} + FARMFROUTPUTCOEFHA_{c} \, (1 - INTENSITYSCALE_{ez,ft}) \end{aligned} \tag{B.286}$$

$$\begin{aligned} managsystoutputcoefha_{ez,ft,mt,c} = & \Bigg[ffmitoutputcoefha_{ez,ft,c} \, MAPTMITFF_{mt} \\ & + \sum_{mofa} (MAPTOMANGSYST_{ez,ft,mofa} \, MODFARMOUTPUTCOEFHA_{mt,mofa,c}) \Bigg] \\ & \times landintensityscalar2_{ez,ft} + FARMFROUTPUTCOEFHA_{c} \, forestryintensityscalar_{ez,ft} \end{aligned} \tag{B.287}$$

$$\begin{aligned} fioutput_{IOag,c} = & \Bigg[\sum_{ez,ft,mt} (managsystoutputcoefha_{ez,ft,mt,c} \, \mathbf{Landuse}_{ez,ft,mt} \, MAPTOIOAG_{IOag,ft}) \\ & + RESIDUALOUTPUT_{IOag,c} \Bigg] \sum_{i} (mfpagdjusted_{dr=DReg1,i} \, INDMAP_{i,IOag}) \end{aligned} \tag{B.288}$$

$$\begin{aligned} fsreturnsperha_{ez,ft,mt} = & \Bigg[\sum_{c} \big(managsystoutputcoefha_{ez,ft,mt,c} \, \mathbf{Pcompcomms}_{sr=SReg1,c} \, prodadjust_{ft} \\ & - mangsystinputcoefha_{ez,ft,mt,c} \, \mathbf{Pcompcommd}_{dr=DReg1,c} \big) \\ & - managsystlabcoefha_{ez,ft,mt} \, pfarmlabour_{ft} \\ & - mangsystdepreccoefha_{ez,ft,mt} \, pfarmbuiltcapital_{ft} \\ & - mangsystindtaxcoefha_{ez,ft,mt} \left(\frac{\mathbf{Gdpindex}}{1000} \right) \Bigg] \, LNDREVENUESH_{ft} \end{aligned} \tag{B.289}$$

$$viableffmit_{ez,ft,mt} = VIABLEMIT_{ez,ft}\, MAPTIMITFF_{mt} + (1 - MAPTMITFF_{mt}) \tag{B.290}$$

$$shareofadoptions_{ez,ft,mt} = \frac{selectedfs_{ez,ft,mt}}{\sum_{mt} selectedfs_{ez,ft,mt}} \tag{B.291}$$

$$farmcontragindreturns_{ez,ft,agi} = \frac{maxpermittedreturns_{ez,ft}\, permittedland_{ez,ft}\, MAPTOAGIND_{ft,agi}}{\sum_{ft} (permittedland_{ez,ft}\, MAPTOAGIND_{ft,agi})} \tag{B.292}$$

$$expagindreturnsperha_{ez,agi} = \sum_{ft} farmcontragindreturns_{ez,ft,agi} \tag{B.293}$$

$$avgreturnsperha_{ez} = \sum_{ft} farmcontrzonereturns_{ez,ft} \tag{B.294}$$

$$\phi^{fland}_{ez} = \frac{\epsilon^{fland}_{ez} + 1}{\epsilon^{fland}_{ez}} \tag{B.295}$$

$$agindshareofallocation1st_{ez,agi} = \left[\frac{(\gamma^{fland}_{ez})^{\phi^{fland}_{ez}}\, \delta^{fland}_{ez,agi}\, avgreturnsperha_{ez}\, expagindreturnsperha_{ez,agi}}{RETURNSCALE_{ez,agi}} \right]^{\frac{1}{1-\phi^{fland}_{ez}}} \tag{B.296}$$

$$agindshareofallocation_{ez,agi} = \frac{agindshareofallocation1st_{ez,agi}}{\sum_{agi} agindshareofallocation1st_{ez,agi}} \tag{B.297}$$

$$permittedland_{ez,ft} = \sum_{mt} \mathbf{Landuse}_{ez,ft,mt} \sum_{mt} shareofadoptions_{ez,ft,mt} \tag{B.298}$$

$$fssharealloc ation_{ez,ft,mt,agi} = \frac{\mathbf{Landuse}_{ez,ft,mt}\, permittednow_{ez,ft,mt}\, MAPTOAGIND_{ft,agi}}{\sum_{ft,mt} (\mathbf{Landuse}_{ez,ft,mt}\, permittednow_{ez,ft,mt}\, MAPTOAGIND_{ft,agi})} \tag{B.299}$$

$$landnomitigation_{ez,ft} = (1 - \sum_{mt} shareofadoptions_{ez,ft,mt})\, totalfslandmitigated_{ez,ft} \tag{B.300}$$

$$landtoreallocate_{ez} = \sum_{ft,mt} landusechangeout_{ez,ft,mt} + \sum_{ft} landnomitigation_{ez,ft} \tag{B.301}$$

$$totallanduse_{ez,agi} = \sum_{ft,mt} (\mathbf{Landuse}_{ez,ft,mt}\, MAPTOAGIND_{ft,agi}) \tag{B.302}$$

$$farmcontrzonereturns_{ez,ft} = \frac{maxpermittedreturns_{ez,ft}\, permittedland_{ez,ft}}{\sum_{ft} permittedland_{ez,ft}} \tag{B.303}$$

$$farmindustryland_{ft} = \sum_{ez,mt} \mathbf{Landuse}_{ez,ft,mt} \tag{B.304}$$

$$landshare_{ft,mt,agi} = \frac{\mathbf{Landuse}_{ez,ft,mt}\, MAPTOAGIND_{ft,agi}}{\sum_{ft,mt}(\mathbf{Landuse}_{ez,ft,mt}\, MAPTOAGIND_{ft,agi})} \tag{B.305}$$

$$\begin{aligned} netlandusechange_{ez,ft} = &\sum_{mt}(landusechangein_{ez,ft,mt} - landusechangeout_{ez,ft,mt} \\ &+ mitigationin_{ez,ft,mt} - mitigationout_{ez,ft,mt}) \end{aligned} \tag{B.306}$$

$$landusechangein_{ez,ft,mt} = \begin{cases} \sum_{agi}(landtoreallocate_{ez}\, agindshareofallocation_{ez,agi} \\ \quad \times fsshareallocation_{ez,ft,mt,agi}) & \text{if} \quad Time \geq KNOWLANDTIME \\ 0 & \text{if} \quad Time < KNOWLANDTIME \end{cases} \tag{B.307}$$

$$totalfslandmitigated_{ez,ft} = \sum_{mt} mitigationout_{ez,ft,mt} \tag{B.308}$$

$$mitigationin_{ez,ft,mt} = \begin{cases} totalfslandmitigated_{ez,ft}\, shareofadoptions_{ez,ft,mt} & \text{if} \quad Time \geq KNOWLANDTIME \\ 0 & \text{if} \quad Time < KNOWLANDTIME \end{cases} \tag{B.309}$$

$$farmretirement_{ez,ft,mt} = \begin{cases} \sum_{agi}(FARMRETIREMENTBYTIME_{ez,agi}(t)\, MAPTOAGIND_{ft,agi}) \\ \quad \times \sum_{agi} landshare_{ez,ft,mt,agi} & \text{if} \quad Time \geq KNOWLANDTIME \\ 0 & \text{if} \quad Time < KNOWLANDTIME \end{cases} \tag{B.310}$$

$$mitigationout_{ez,ft,mt} = \begin{cases} \max[0, (\mathbf{Landuse}_{ez,ft,mt} - LANDUSERESTRICTION_{ez,ft,mt}) \\ \quad \times adoptionrate_{ez,ft,mt}\, deadlinepressure_{ez,ft,mt}] & \text{if} \quad Time \geq KNOWLANDTIME \\ 0 & \text{if} \quad Time < KNOWLANDTIME \end{cases} \tag{B.311}$$

$$fencingcostsbytimeai_{fmu,agi} = \begin{cases} ANNUALFENCE_{fmu,agi} & \text{if} \quad Time > FENCESTART_{fmu,agi} \\ & \text{and} \quad Time \leq FENCEEND_{fmu,agi} \\ 0 & \text{if} \quad Time \leq FENCESTART_{fmu,agi} \\ & \text{and} \quad Time > FENCEEND_{fmu,agi} \end{cases} \tag{B.312}$$

$$\begin{aligned} fencingcostsbytimeioag_{fmu,IOag=IOAg01} = &fencingcostsbytimeai_{fmu,agi=AgIn01} \\ &+ fencingcostsbytimeai_{fmu,agi=AgIn02} + fencingcostsbytimeai_{fmu,agi=AgIn05} \\ &+ fencingcostsbytimeai_{fmu,agi=AgIn06} \\ fencingcostsbytimeioag_{fmu,IOag=IOAg02} = &fencingcostsbytimeai_{fmu,agi=AgIn03} \\ fencingcostsbytimeioag_{fmu,IOag=IOAg03} = &fencingcostsbytimeai_{fmu,agi=AgIn04} \end{aligned} \tag{B.313}$$

$$plancost1bytime_{fmu,agi} = FARMPERHA_{fmu,agi} \sum_{ez} (totallanduse_{ez,agi}\, FMUZONEMAP_{fmu,ez}) \times \left[\frac{FIRSTPCONSENTC_{agi}\, initialpyear_{agi} + ANPCONSENTC_{agi}\, subpyear_{agi}}{1000000} \right] \tag{B.314}$$

$$plancost2bytime_{fmu,agi} = FARMPERHA_{fmu,agi} \sum_{ez} (totallanduse_{ez,agi}\, FMUZONEMAP_{fmu,ez}) \times \left[\frac{FIRSTPOTHERC_{agi}\, initialpyear_{agi} + ANPOTHERC_{agi}\, subpyear_{agi}}{1000000} \right] \tag{B.315}$$

$$\begin{aligned}
plancostsbytimeioag_{fmu,IOag=IOAg01} &= councilfeeprice\,(plancost1bytime_{fmu,agi=Ag01} \\
&\quad + plancost1bytime_{fmu,agi=Ag02} + plancost1bytime_{fmu,agi=Ag05} \\
&\quad + plancost1bytime_{fmu,agi=Ag06} + councilfee1_{fmu,IOag=IOAg01}) \\
&\quad + otherfeeprice\,(plancost2bytime_{fmu,agi=Ag01} \\
&\quad + plancost2bytime_{fmu,agi=Ag02} + plancost2bytime_{fmu,agi=Ag05} \\
&\quad + plancost2bytime_{fmu,agi=Ag06} + plancost2bytime_{fmu,agi=Ag01} \\
&\quad + otherfee1_{fmu,IOag=IOAg01}) \\
plancostsbytimeioag_{fmu,IOag=IOAg02} &= councilfeeprice\,(plancost1bytime_{fmu,agi=Ag03} \\
&\quad + councilfee1_{fmu,IOag=IOAg02}) + otherfeeprice \\
&\quad \times (plancost2bytime_{fmu,agi=Ag03} + otherfee1_{fmu,IOag=IOAg02}) \\
plancostsbytimeioag_{fmu,IOag=IOAg03} &= councilfeeprice\,(plancost1bytime_{fmu,agi=Ag04} \\
&\quad + councilfee1_{fmu,IOag=IOAg03}) + otherfeeprice \\
&\quad \times (plancost2bytime_{fmu,agi=Ag04} + otherfee1_{fmu,IOag=IOAg03})
\end{aligned} \tag{B.316}$$

$$maxpermittedreturns_{ez,ft} = \max_{mt}[fspermittedreturns_{ez,ft,mt}] \tag{B.317}$$

$$initialpyear_{agi} = \begin{cases} 1 & \text{if} \quad FIRSTPTIME_{agi} + 0.5 > Time > FIRSTPTIME_{agi} - 0.5 \\ 0 & \text{if} \quad FIRSTPTIME_{agi} + 0.5 \leq Time \leq FIRSTPTIME_{agi} - 0.5 \end{cases} \tag{B.318}$$

$$subpyear_{agi} = \begin{cases} 1 & \text{if} \quad Time > FIRSTPTIME_{agi} + 0.5 \\ 0 & \text{if} \quad Time \leq FIRSTPTIME_{agi} + 0.5 \end{cases} \tag{B.319}$$

$$belowcap_{ez,ft,mt} = \begin{cases} 1 & \text{if} \quad \mathbf{Landuse}_{ez,ft,mt} < LANDUSERESTRICTION_{ez,ft,mt} - 0.001 \\ 0 & \text{if} \quad \mathbf{Landuse}_{ez,ft,mt} \geq LANDUSERESTRICTION_{ez,ft,mt} - 0.001 \end{cases} \tag{B.320}$$

$$fspermittedreturns_{ez,ft,mt} = \begin{cases} \mathbf{Percfsreturnsperha}_{ez,ft,mt} & \text{if} \quad belowcap_{ez,ft,mt} = 1 \text{ and } viableffmit_{ez,ft,mt} = 1 \\ & \quad \text{and} \quad \mathbf{Percfsreturnsperha}_{ez,ft,mt} > 0 \\ -1000 & \text{if} \quad belowcap_{ez,ft,mt} = 0 \text{ or } viableffmit_{ez,ft,mt} = 0 \\ & \quad \text{or} \quad \mathbf{Percfsreturnsperha}_{ez,ft,mt} = 0 \end{cases} \tag{B.321}$$

$$rulenotified_{ez,ft,mt} = \begin{cases} 1 & \text{if} \quad Time \geq NONPERMITNOTICEDATE_{ez,ft,mt} \\ 0 & \text{if} \quad Time < NONPERMITNOTICEDATE_{ez,ft,mt} \end{cases} \tag{B.322}$$

$$permittednow_{ez,ft,mt} = \begin{cases} 1 & \text{if} \quad rulenotified_{ez,ft,mt} = 0 \quad \text{and} \\ & \qquad \mathbf{Landuse}_{ez,ft,mt} - LANDUSERESTRICTION_{ez,ft,mt} \leq 0 \\ 0 & \text{if} \quad rulenotified_{ez,ft,mt} = 1 \quad \text{and} \\ & \qquad \mathbf{Landuse}_{ez,ft,mt} - LANDUSERESTRICTION_{ez,ft,mt} > 0 \end{cases} \tag{B.323}$$

$$adoptiontype_{ez,ft,mt} = \begin{cases} SELECTEDADOPTION_{ez,ft,mt} & \text{if} \quad rulenotified_{ez,ft,mt} = 0 \\ 0 & \text{if} \quad rulenotified_{ez,ft,mt} = 1 \end{cases} \tag{B.324}$$

$$selectedfs_{ez,ft,mt} = \begin{cases} 1 & \text{if} \quad fspermittedreturns_{ez,ft,mt} - maxpermittedreturns_{ez,ft} \geq -0.00000001 \;\text{ and} \\ & \quad 1 - rulenotified_{ez,ft,mt} + belowcap_{ez,ft,mt} > 0 \;\text{ and} \\ & \quad fspermittedreturns \neq -1000 \\ 0 & \text{if} \quad fspermittedreturns_{ez,ft,mt} - maxpermittedreturns_{ez,ft} < -0.00000001 \;\text{ or} \\ & \quad 1 - rulenotified_{ez,ft,mt} + belowcap_{ez,ft,mt} \leq 0 \;\text{ or} \\ & \quad fspermittedreturns \neq -1000 \end{cases} \tag{B.325}$$

$$landusechangeout_{ez,ft,mt} = \begin{cases} \mathbf{Landuse}_{ez,ft,mt} MAXCHANGERT_{ft} & \text{if} \quad Time \geq KNOWLANDTIME \quad \text{and} \\ & \qquad Time \leq HOLDLANDTIME \\ 0 & \text{if} \quad Time < KNOWLANDTIME \quad \text{or} \\ & \qquad Time > HOLDLANDTIME \end{cases} \tag{B.326}$$

$$noticeperiod_{ez,ft,mt} = DATENONPERMITTED_{ez,ft,mt} - NONPERMITNOTICEDATE_{ez,ft,mt} \tag{B.327}$$

$$midpoint_{ez,ft,mt} = NONPERMITNOTICEDATE_{ez,ft,mt} + noticeperiod_{ez,ft,mt}\, noticeperiodscalar \tag{B.328}$$

$$noticeperiodscalar = \begin{cases} 0.5 & \text{if} \quad adoptiontype_{ez,ft,mt} = 1 \\ 0.85 & \text{if} \quad adoptiontype_{ez,ft,mt} = 2 \\ 1.25 & \text{if} \quad adoptiontype_{ez,ft,mt} = 3 \end{cases} \tag{B.329}$$

$$adoptionsteepness_{ez,ft,mt} = \begin{cases} \frac{4}{noticeperiod_{ez,ft,mt}} & \text{if} \quad adoptiontype_{ez,ft,mt} = 1 \\ \frac{6}{noticeperiod_{ez,ft,mt}} & \text{if} \quad adoptiontype_{ez,ft,mt} = 2 \\ \frac{8}{noticeperiod_{ez,ft,mt}} & \text{if} \quad adoptiontype_{ez,ft,mt} = 3 \end{cases} \tag{B.330}$$

$$adoptionrate_{ez,ft,mt} = \frac{MAXIMUMRATE_{ez,ft,mt}}{1 + e^{(-adoptionsteepness_{ez,ft,mt})\,(Time - midpoint_{ez,ft,mt})}} \tag{B.331}$$

$$deadlinepressure_{ez,ft,mt} = \min\left[20, 5\left(\frac{1}{DATENONPERMITTED_{ez,ft,mt} - Time} - \frac{1}{noticeperiod_{ez,ft,mt}}\right)\right] \tag{B.332}$$

$$\begin{aligned} sharefiinputdisag_{IOag,input=InterI} &= \mathbf{Pintinputs}_{dr=DReg1,i\to IOag}\left(fifiinputcoef_{IOag,input=InterI}\right)^{1-\eta^{fifiinput}_{IOag}} \\ sharefiinputdisag_{IOag,input=FactsI} &= \mathbf{Pcfact}_{dr=DReg1,i\to IOag}\left(fifiinputcoef_{IOag,input=FactsI}\right)^{1-\eta^{fifiinput}_{IOag}} \end{aligned} \tag{B.333}$$

$$\begin{aligned} sharecapitaldisag_{IOag,cap=BuilC} &= \mathbf{Pbuiltcap}_{dr,i\to IOag}\left(ficapitalinput_{IOag,cap=BuilC}\right)^{1-\eta^{ficapital}_{IOag}} \\ sharecapitaldisag_{IOag,cap=LandC} &= \mathbf{Pcompnaturalcapd}_{dr,i\to IOag}\left(ficapitalinput_{IOag,cap=LandC}\right)^{1-\eta^{ficapital}_{IOag}} \end{aligned} \tag{B.334}$$

$$fisharecominput_{IOag,c} = \frac{\mathbf{Pcompcommd}_{dr=DReg1,c}\left(fiintinputcoef_{IOag,c}\right)^{1-\eta^{ficominput}_{IOag}}}{\sum_c\left[\mathbf{Pcompcommd}_{dr=DReg1,c}\left(fiintinputcoef_{IOag,c}\right)^{1-\eta^{ficominput}_{IOag}}\right]} \tag{B.335}$$

$$fisharecomsup_{IOag,c} = \frac{(\mathbf{Pcompcomms}_{sr=SReg1,c}\left(fioutput_{IOag,c}\right)^{1-\phi^{comoutput}_{IOag}}}{\sum_c\left[\mathbf{Pcompcomms}_{sr=SReg1,c}\left(fioutput_{IOag,c}\right)^{1-\phi^{comoutput}_{IOag}}\right]} \tag{B.336}$$

$$fisharefiinput_{IOag,input} = \frac{sharefiinputdisag_{IOag,input}}{\sum_{input} sharefiinputdisag_{IOag,input}} \tag{B.337}$$

$$fisharefactorinput_{IOag,h} = \frac{\left(fifactorinput_{IOag,h}\right)^{1-\eta^{fifactinput}_{IOag}}}{\sum_h\left[\left(fifactorinput_{IOag,h}\right)^{1-\eta^{fifactinput}_{IOag}}\right]} \tag{B.338}$$

$$fisharecapitalinput_{IOag,cap} = \frac{sharecapitaldisag_{IOag,cap}}{\sum_{cap} sharecapitaldisag_{IOag,cap}} \tag{B.339}$$

$$fisalecominput_{IOag} = \frac{1}{\sum_c\left[\left(fisharecominput_{IOag,c}\;\left(fiintinputcoef_{IOag,c}\right)^{\eta^{ficominput}_{IOag}}\right)^{\frac{1}{\eta^{ficominput}_{IOag}}}\right]} \tag{B.340}$$

$$fiscalefiinput_{IOag} = \frac{1}{\sum_{input}\left[\left(fisharefiinput_{IOag,input}\;\left(fifiinputcoef_{IOag,input}\right)^{\eta^{fifiinput}_{IOag}}\right)^{\frac{1}{\eta^{fifiinput}_{IOag}}}\right]} \tag{B.341}$$

$$fiscalefactorinput_{IOag} = \frac{\sum_h fifactorinput_{IOag,h}}{\sum_h \left[\left(fisharefactorinput_{IOag,h} \; (fifactorinput_{IOag,h})^{\eta^{fifactinput}_{IOag}} \right)^{\frac{1}{\eta^{fifactinput}_{IOag}}} \right]} \quad (B.342)$$

$$fiscalecapitalinput_{IOag} = \frac{\sum_{cap} ficapitalinput_{IOag,cap}}{\sum_{cap} \left[\left(fisharecapitalinput_{IOag,cap} \; (ficapitalinput_{IOag,cap})^{\eta^{ficapital}_{IOag}} \right)^{\frac{1}{\eta^{ficapital}_{IOag}}} \right]} \quad (B.343)$$

$$\begin{aligned} totalplandemand_{dr=DReg1,c} &= PLANMAP1_c \left[\sum_{fmu} \left(\sum_{agi} plancost1bytime_{fmu,agi} \right.\right. \\ &\quad \left.\left. + \sum_{IOag} councilfee1_{fmu,IOag} \right) \right] \\ &\quad + PLANMAP2_c \left[\sum_{fmu} \left(\sum_{agi} plancost2bytime_{fmu,agi} \right.\right. \\ &\quad \left.\left. + \sum_{IOag} otherfee1_{fmu,IOag} \right) \right] \\ totalplandemand_{dr=DReg2,c} &= 0 \end{aligned} \quad (B.344)$$

$$\begin{aligned} totalfencingdemand_{dr=DReg1,c} &= \sum_{fmu} (fencingcostsbytimeioag_{fmu,IOag}) \, FENCEMAP_c \\ totalfencingdemand_{dr=DReg2,c} &= 0 \end{aligned} \quad (B.345)$$

$$indfencingcosts_{fmu,i} = \sum_{IOag} (fencingcostsbytimeioag_{fmu,IOag} \, INDMAP_{i,IOag}) \; fenceprice \quad (B.346)$$

$$indplancosts_{fmu,i} = \sum_{IOag} (plancostsbytimeioag_{fmu,IOag} \, INDMAP_{i,IOag}) \quad (B.347)$$

$$fiscalecomsup_{IOag} = \frac{\sum_c fioutput_{IOag,c}}{\sum_c \left[\left(fisharecomsup_{IOag,c} \; (fioutput_{IOag,c})^{\phi^{comoutput}_{IOag}} \right)^{\frac{1}{\phi^{comoutput}_{IOag}}} \right]} \quad (B.348)$$

$$industryland_{IOag} = \sum_{ft} \left(MAPTOIOAG_{IOag,ft} \sum_{ez,mt} \mathbf{Landuse}_{ez,ft,mt} \right) \quad (B.349)$$

$$netratelandusechange_{IOag} = \frac{\sum_{ez,ft} (netlandusechange_{ez,ft} \, MAPTOIOAG_{IOag,ft})}{industryland_{IOag}} \quad (B.350)$$

$$\begin{aligned} agriprodrt_{IOag=IOAg01} &= \frac{\sum_c fioutput_{IOag=IOAg01,c} \left(\sum_{ez,ft,mt} iLanduse_{ez,ft,mt} \, MAPTOIOAG_{IOag=IOAg01,ft} \right)}{\left(\sum_{ez,ft,mt} \mathbf{Landuse}_{ez,ft,mt} \, MAPTOIOAG_{IOag=IOAg01,ft} \right) iIndustryaccount_{i=Ind1,dr=DReg1}} \\ agriprodrt_{IOag=IOAg02} &= \frac{\sum_c fioutput_{IOag=IOAg02,c} \left(\sum_{ez,ft,mt} iLanduse_{ez,ft,mt} \, MAPTOIOAG_{IOag=IOAg02,ft} \right)}{\left(\sum_{ez,ft,mt} \mathbf{Landuse}_{ez,ft,mt} \, MAPTOIOAG_{IOag=IOAg02,ft} \right) iIndustryaccount_{i=Ind2,dr=DReg1}} \\ agriprodrt_{IOag=IOAg03} &= \frac{\sum_c fioutput_{IOag=IOAg03,c} \left(\sum_{ez,ft,mt} iLanduse_{ez,ft,mt} \, MAPTOIOAG_{IOag=IOAg03,ft} \right)}{\left(\sum_{ez,ft,mt} \mathbf{Landuse}_{ez,ft,mt} \, MAPTOIOAG_{IOag=IOAg03,ft} \right) iIndustryaccount_{i=Ind3,dr=DReg1}} \end{aligned} \quad (B.351)$$

$$\begin{aligned}\eta^{ficapital}_{IOag=IOAg01} &= \eta^{cc}_{dr=DReg1,i=Ind1\rightarrow IOag}\\ \eta^{ficapital}_{IOag=IOAg02} &= \eta^{cc}_{dr=DReg1,i=Ind2\rightarrow IOag}\\ \eta^{ficapital}_{IOag=IOAg03} &= \eta^{cc}_{dr=DReg1,i=Ind3\rightarrow IOag}\end{aligned} \tag{B.352}$$

$$\begin{aligned}\eta^{ficominput}_{IOag=IOAg01} &= \eta^{cominput}_{dr=DReg1,i=Ind1\rightarrow IOag}\\ \eta^{ficominput}_{IOag=IOAg02} &= \eta^{cominput}_{dr=DReg1,i=Ind2\rightarrow IOag}\\ \eta^{ficominput}_{IOag=IOAg03} &= \eta^{cominput}_{dr=DReg1,i=Ind3\rightarrow IOag}\end{aligned} \tag{B.353}$$

$$\begin{aligned}\eta^{fifactinput}_{IOag=IOAg01} &= \eta^{fact}_{dr=DReg1,i=Ind1\rightarrow IOag}\\ \eta^{fifactinput}_{IOag=IOAg02} &= \eta^{fact}_{dr=DReg1,i=Ind2\rightarrow IOag}\\ \eta^{fifactinput}_{IOag=IOAg03} &= \eta^{fact}_{dr=DReg1,i=Ind3\rightarrow IOag}\end{aligned} \tag{B.354}$$

$$\begin{aligned}\eta^{fiffiinput}_{IOag=IOAg01} &= \eta^{fi}_{dr=DReg1,i=Ind1\rightarrow IOag}\\ \eta^{fiffiinput}_{IOag=IOAg02} &= \eta^{fi}_{dr=DReg1,i=Ind2\rightarrow IOag}\\ \eta^{fiffiinput}_{IOag=IOAg03} &= \eta^{fi}_{dr=DReg1,i=Ind3\rightarrow IOag}\end{aligned} \tag{B.355}$$

$$\begin{aligned}\phi^{comoutput}_{IOag=IOAg01} &= \phi^{comsup}_{sr=SReg1,i=Ind1\rightarrow IOag}\\ \phi^{comoutput}_{IOag=IOAg02} &= \phi^{comsup}_{sr=SReg1,i=Ind2\rightarrow IOag}\\ \phi^{comoutput}_{IOag=IOAg03} &= \phi^{comsup}_{sr=SReg1,i=Ind3\rightarrow IOag}\end{aligned} \tag{B.356}$$

$$\begin{aligned}prodadjust_{ft=FaTyp01} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp02} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp03} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp04} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp05} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp06} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp07} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp08} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp09} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp10} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp11} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp12} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp13} &= mfpadjusted_{dr=DReg1,i=Ind1}\\ prodadjust_{ft=FaTyp14} &= mfpadjusted_{dr=DReg1,i=Ind2}\end{aligned}$$

$$\begin{aligned}
prodadjust_{ft=FaTyp15} &= mfpadjusted_{dr=DReg1,i=Ind3} \\
prodadjust_{ft=FaTyp16} &= mfpadjusted_{dr=DReg1,i=Ind1} \\
prodadjust_{ft=FaTyp17} &= mfpadjusted_{dr=DReg1,i=Ind1}
\end{aligned} \tag{B.357}$$

$$\begin{aligned}
pfarmbuiltcapital_{ft=FaTyp01} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp02} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp03} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp04} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp05} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp06} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp07} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp08} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp09} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp10} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp11} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp12} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp13} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp14} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind2} \\
pfarmbuiltcapital_{ft=FaTyp15} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind3} \\
pfarmbuiltcapital_{ft=FaTyp16} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1} \\
pfarmbuiltcapital_{ft=FaTyp17} &= \mathbf{Pbuiltcap}_{dr=DReg1,i=Ind1}
\end{aligned} \tag{B.358}$$

$$\begin{aligned}
pfarmlabour_{ft=FaTyp01} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp02} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp03} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp04} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp05} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp06} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp07} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp08} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp09} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp10} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1}
\end{aligned}$$

$$\begin{aligned}
pfarmlabour_{ft=FaTyp11} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp12} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp13} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp14} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind2} \\
pfarmlabour_{ft=FaTyp15} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind3} \\
pfarmlabour_{ft=FaTyp16} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1} \\
pfarmlabour_{ft=FaTyp17} &= \mathbf{Pfact}_{h=LAB,dr=DReg1,i=Ind1}
\end{aligned} \tag{B.359}$$

$$\begin{aligned}
modfarmitffinputcoefha_{mofa=MoFa01,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa01,c} \\
modfarmitffinputcoefha_{mofa=MoFa02,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa02,c} \\
modfarmitffinputcoefha_{mofa=MoFa03,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa03,c} \\
modfarmitffinputcoefha_{mofa=MoFa04,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa04,c} \\
modfarmitffinputcoefha_{mofa=MoFa05,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa05,c} \\
modfarmitffinputcoefha_{mofa=MoFa06,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa06,c} \\
modfarmitffinputcoefha_{mofa=MoFa07,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa07,c} \\
modfarmitffinputcoefha_{mofa=MoFa08,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa08,c} \\
modfarmitffinputcoefha_{mofa=MoFa09,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa09,c} \\
modfarmitffinputcoefha_{mofa=MoFa10,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa10,c} \\
modfarmitffinputcoefha_{mofa=MoFa11,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa11,c} \\
modfarmitffinputcoefha_{mofa=MoFa12,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa12,c} \\
modfarmitffinputcoefha_{mofa=MoFa13,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa13,c} \\
modfarmitffinputcoefha_{mofa=MoFa14,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa14,c} \\
modfarmitffinputcoefha_{mofa=MoFa15,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa15,c} \\
modfarmitffinputcoefha_{mofa=MoFa16,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa16,c} \\
modfarmitffinputcoefha_{mofa=MoFa17,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa17,c} \\
modfarmitffinputcoefha_{mofa=MoFa18,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa18,c} \\
modfarmitffinputcoefha_{mofa=MoFa19,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa19,c} \\
modfarmitffinputcoefha_{mofa=MoFa20,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa20,c} \\
modfarmitffinputcoefha_{mofa=MoFa21,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa21,c} \\
modfarmitffinputcoefha_{mofa=MoFa22,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa22,c} \\
modfarmitffinputcoefha_{mofa=MoFa23,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa23,c} \\
modfarmitffinputcoefha_{mofa=MoFa24,c} &= MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa24,c}
\end{aligned}$$

$$modfarmitffinputcoefha_{mofa=MoFa25,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa25,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa26,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa26,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa27,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa27,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa28,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa28,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa29,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa29,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa30,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa30,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa31,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa31,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa32,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa32,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa33,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa33,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa34,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa34,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa35,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa35,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa36,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa36,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa37,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa37,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa38,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa38,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa39,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa39,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa40,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa40,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa41,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa41,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa42,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa42,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa43,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa43,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa44,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa44,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa45,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa45,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa46,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa46,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa47,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa47,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa48,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa48,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa49,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa49,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa50,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa50,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa51,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa51,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa52,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa52,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa53,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa53,c}$$
$$modfarmitffinputcoefha_{mofa=MoFa54,c} = MODFARMINPUTCOEFHA_{mt=Miti04,mofa=MoFa54,c}$$

$$modfarmitffinputcoefha_{mofa=MoFa55,c} = 0$$
$$modfarmitffinputcoefha_{mofa=MoFa56,c} = 0 \quad (B.360)$$
$$modfarmitffinputcoefha_{mofa=MoFa57,c} = 0$$

$$modfarmmitffoutputcoefha_{mofa=MoFa01,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa01,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa02,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa02,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa03,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa03,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa04,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa04,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa05,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa05,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa06,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa06,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa07,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa07,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa08,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa08,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa09,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa09,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa10,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa10,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa11,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa11,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa12,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa12,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa13,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa13,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa14,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa14,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa15,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa15,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa16,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa16,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa17,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa17,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa18,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa18,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa19,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa19,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa20,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa20,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa21,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa21,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa22,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa22,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa23,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa23,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa24,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa24,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa25,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa25,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa26,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa26,c}$$
$$modfarmmitffoutputcoefha_{mofa=MoFa27,c} = MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa27,c}$$

$$
\begin{aligned}
modfarmmitffoutputcoefha_{mofa=MoFa28,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa28,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa29,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa29,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa30,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa30,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa31,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa31,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa32,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa32,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa33,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa33,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa34,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa34,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa35,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa35,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa36,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa36,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa37,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa37,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa38,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa38,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa39,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa39,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa40,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa40,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa41,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa41,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa42,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa42,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa43,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa43,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa44,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa44,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa45,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa45,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa46,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa46,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa47,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa47,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa48,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa48,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa49,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa49,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa50,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa50,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa51,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa51,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa52,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa52,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa53,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa53,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa54,c} &= MODFARMOUTPUTCOEFHA_{mt=Miti04,mofa=MoFa54,c} \\
modfarmmitffoutputcoefha_{mofa=MoFa55,c} &= 0 \\
modfarmmitffoutputcoefha_{mofa=MoFa56,c} &= 0 \\
modfarmmitffoutputcoefha_{mofa=MoFa57,c} &= 0
\end{aligned} \tag{B.361}
$$

B.12 Rest of world module equations

B.12.1 Stocks

$$\frac{d}{dt}(\mathbf{Exchangert}) = \begin{cases} \frac{1}{TIME\ STEP}\left(ACTUALEXCHANGERT - \mathbf{Exchangert}\right) & \text{for} \quad t < 3 \\ \left(\left(\frac{1}{bopratio}\right)^{\alpha^{exchangert}} - 1\right)\mathbf{Exchangert} & \text{for} \quad t \geq 3 \end{cases} \tag{B.362}$$

B.12.2 Auxiliaries

$$bopratio = \frac{rwincome}{rwexpenditure} \tag{B.363}$$

$$\begin{aligned} rwincome = \sum_{dr} \Bigg(& rwlabourincome_{dr} + \sum_{c}\left(nzimportpurchases_{dr,c}\right) + entrwtrans_{dr} \\ & + hhldrwtrans_{dr} + \sum_{g}\left(govtrwtrans_{g,dr}\right) - rwdirecttax_{dr}\Bigg) \end{aligned} \tag{B.364}$$

$$rwlabourincome_{dr} = \sum_{i}(rwlaboursupply_{dr,i}\, \mathbf{Pfact}_{h=LAB,dr,i}) \tag{B.365}$$

$$rwlaboursupply_{dr,i} = factorsu_{h=LAB,dr,i}\,\frac{RWFACTRT_{h=LAB,dr}}{1 - RWFACTRT_{h=LAB,dr}} \tag{B.366}$$

$$nzimportpurchases_{dr,c} = importdemand_{dr,c}\,\frac{PCOMMWORLDIMP_c(t)}{\mathbf{Exchangert}} \tag{B.367}$$

$$rwdirecttax_{dr} = (entrwtrans_{dr} + rwlabourincome_{dr})\,RWDIRECTTAXRT_{dr} \tag{B.368}$$

$$\begin{aligned} rwexpenditure = & \sum_{dr}\left(rwsavings_{dr} + rwenttrans_{dr} + rwhhldtrans_{dr} + rwindirecttax_{dr}\right) \\ & + \sum_{sr}\sum_{c}(nzexportsales_{sr,c}) \end{aligned} \tag{B.369}$$

$$rwindirecttax_{dr} = \sum_{c}(nzexportsales_{sr\rightarrow dr,c})\,RWINDIRECTTAXRT_{dr} \tag{B.370}$$

$$nzexportsales_{sr,c} = \frac{pexportcommd_{sr,c}\,actualexports_{sr,c}}{\mathbf{Exchangert}} \tag{B.371}$$

$$actualexports_{sr,c} = \min\left(expcommodityd_{sr,c}, expcommoditys_{sr,c}\right) \tag{B.372}$$

B.13 Output variable module equations

B.13.1 Stocks

$$\frac{d}{dt}\left(\mathbf{Inflationrt}\right) = \frac{1}{\tau}\left(targetinflationrt - \mathbf{Inflationrt}\right) \tag{B.373}$$

$$\frac{d}{dt}\left(\mathbf{Holdregvaladd}_{dr}\right) = \frac{1}{\tau}\left(totalvalueadded_{dr} - \mathbf{Holdregvaladd}_{dr}\right) \tag{B.374}$$

B.13.2 Auxiliaries

$$targetinflationrt = \begin{cases} ACINFLATIONRT(t) & \text{for} \quad t < 11.5 \\ desiredinflationrt & \text{for} \quad t \geq 11.5 \end{cases} \tag{B.375}$$

$$desiredinflationrt = 4\left(\frac{cpif(t) - cpif(t-0.25)}{cpif(t-0.25)}\right) \tag{B.376}$$

$$cpif = \sqrt{cpip \times cpil} \tag{B.377}$$

$$cpil = 1000\,\frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\,BASEHHLDCONSUMP_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\,BASEHHLDCONSUMP_{dr,c}\right)} \tag{B.378}$$

$$cpip = 1000\,\frac{\sum_{dr}\sum_{c}\left(\mathbf{Pcompcommd}_{dr,c}\,hhldconsump_{dr,c}\right)}{\sum_{dr}\sum_{c}\left(BASEPCOMPCOMMD_{dr,c}\,hhldconsump_{dr,c}\right)} \tag{B.379}$$

$$actualgdpindex = 1000 \times gdpindexp \tag{B.380}$$

$$\begin{aligned} gdpindexp = {} & \Bigg[\sum_{dr,c}\Bigg(\Bigg(hhldconsump_{dr,c} + \sum_{g} govtconsump_{g,dr,c} + investconsumpq_{dr,c}\Bigg)\,\mathbf{Pcompcommd}_{dr,c} \\ & + stockchanges_{dr,c}\Bigg) + \sum_{sr,c} nzexportsales_{sr,c} - \sum_{dr,c} nzimportpurchases_{dr,c}\Bigg] \\ & \div \\ & \Bigg[\sum_{dr,c}\Bigg(\Bigg(hhldconsump_{dr,c} + \sum_{g} govtconsump_{g,dr,c} + investconsumpq_{dr,c} \\ & + \frac{stockchanges_{dr,c}}{\mathbf{Pcompcommd}_{dr,c}}\Bigg)\,BASECOMPCOMMD_{dr,c}\Bigg) + \sum_{sr,c}\left(\frac{nzexportsales_{sr,c}}{pexportcommd_{sr,c}}\,\mathbf{Exchangert}\right) \\ & - \sum_{dr,c}\left(\frac{nzimportpurchases_{dr,c}}{PCOMMWORLDIMP_{c}(t)}\,\mathbf{Exchangert} \times 1\right)\Bigg] \end{aligned} \tag{B.381}$$

$$stockchanges_{dr,c} = vcommoditysupply_{dr,c} - vcommoditydemand_{dr,c} \tag{B.382}$$

$$realgdp = 1000\frac{totalexpenditure}{actualgdpindex} \tag{B.383}$$

$$gdppercapita = \frac{realgdp}{\sum_{dr} POPULATION_{dr}(t)} \tag{B.384}$$

$$\begin{aligned} totalexpenditure =& \sum_{dr,c}\left[\left(hhldconsump_{dr,c} + \sum_{g} govtconsump_{g,dr,c} + investconsumpq_{dr,c}\right) \mathbf{Pcompcommd}_{dr,c}\right] \\ &+ \sum_{sr,c} nzexportsales_{sr,c} - \sum_{dr,c} nzimportpurchases_{dr,c} \\ &+ \sum_{dr,c} stockchanges_{dr,c} + \sum_{dr} otherindirecttaxes_{dr} \end{aligned} \tag{B.385}$$

$$vcommoditysupply_{dr,c} = \sum_{i}(actualsupplys_{sr\rightarrow dr,i,c}) + vregimports_{dr,c} + nzimportpurchases_{dr,c} \tag{B.386}$$

$$vregimports_{dr,c} = regcdomcomms_{SReg1\leftrightarrow DReg2,SReg2\leftrightarrow DReg1,c}\, Pregdomcomm_{SReg1\leftrightarrow DReg2,SReg2\leftrightarrow DReg1,c} \tag{B.387}$$

$$vregexports_{dr,c} =?? \tag{B.388}$$

$$\begin{aligned} vcommoditydemand_{dr,c} =& vhhldconsump_{dr,c} + \sum_{g}\left(vgovtconsump_{g,dr,c}\right) + vinvestconsump_{dr,c} \\ &+ vregexports_{dr,c} + nzexportsales_{sr\rightarrow dr,c} + totalcommodityexpend_{dr,c} \end{aligned} \tag{B.389}$$

$$vhhldconsump_{dr,c} = hhldconsump_{dr,c}\, \mathbf{Pcompcommd}_{dr,c} \tag{B.390}$$

$$vgovtconsump_{g,dr,c} = govtconsump_{g,dr,c}\, \mathbf{Pcompcommd}_{dr,c} \tag{B.391}$$

$$vinvestconsump_{dr,c} = investconsumpq_{dr,c}\, \mathbf{Pcompcommd}_{dr,c} \tag{B.392}$$

$$totalcommodityexpend_{dr,c} = \sum_{i}(domcommexpend_{dr,i,c} + importcommexpend_{dr,i,c}) \tag{B.393}$$

$$dindvalueadded_{dr,i} = 1000\,\frac{indvalueadded_{dr,i}}{actualgdpindex} \tag{B.394}$$

$$dtotalvalueadded_{dr} = 1000\,\frac{totalvalueadded_{dr}}{actualgdpindex} \tag{B.395}$$

$$\begin{aligned} indvalueadded_{dr,i} =& \sum_{h}\left(factorsu_{h,dr,i}\, \mathbf{Pfact}_{h,dr,i}\right) + realindustrybalance_{dr,i} \\ &+ indindirecttax_{dr,i} \end{aligned} \tag{B.396}$$

$$totalvalueadded_{dr} = \sum_{i} indvalueadded_{dr,i} + otherindirecttaxes_{dr} \tag{B.397}$$

$$\begin{aligned} otherindirecttaxes_{dr} =& \sum_{g} govtindirecttax_{g,dr} + hhldindirecttax_{dr} + rwindirecttax_{dr} \\ &+ investindirecttax_{dr} \end{aligned} \tag{B.398}$$

B.14 Freshwater Management Unit and Territorial Local Authority reporting module equations

B.14.1 Auxiliaries

$$zonefarmlab_{ez,ft} = \sum_{mt} (managsystlabcoefha_{ez,ft,mt}\, pfarmlabour_{ft}\, \mathbf{Landuse}_{ez,ft,mt}) \tag{B.399}$$

$$\begin{aligned} zonefarmcapital_{ez,ft} = \sum_{mt} \Bigg[\left(\frac{fsreturnsperha_{ez,ft,mt}}{LNDREVENUESH_{ft}} + mangsystdepreccoefha_{ez,ft,mt}\, pfarmbuiltcapital_{ft} \right) \\ \times \mathbf{Landuse}_{ez,ft,mt} \Bigg] \end{aligned} \tag{B.400}$$

$$fmufarmlab_{fmu,ft} = \sum_{ez} (zonefarmlab_{ez,ft}\, FMUZONEMAP_{fmu,ez}) \tag{B.401}$$

$$fmufarmlabIOind_{fmu,IOag} = \sum_{ft} (fmufarmlabfmu, ft\, MAPTOIOAG_{IOag,ft}) \tag{B.402}$$

$$\begin{aligned} residualcap_{IOag} = \sum_{i} (\mathbf{Pfact}_{h=CAP,dr=DReg1,i}\, INDMAP_{i,IOag}) \\ \times (RESIDUALBCAPITAL_{IOag} + RESIDUALLAND_{IOag}) \end{aligned} \tag{B.403}$$

$$fmufarmcapital_{fmu,ft} = \sum_{ez} (zonefarmcapital_{ez,ft}\, FMUZONEMAP_{fmu,ez}) \tag{B.404}$$

$$residuallab_{IOag} = \sum_{i} (\mathbf{Pfact}_{h=LAB,dr=DReg1,i}\, INDMAP_{i,IOag})\, RESIDUALLABOUR_{IOag} \tag{B.405}$$

$$\begin{aligned} fmuagindva_{fmu,IOag} = \sum_{ft} \left[MAPTOIOAG_{IOag,ft}(fmufarmlab_{fmu,ft} + fmufarmcapital_{fmu,ft}) \right] \\ + FMUSHRESIDUAL_{fmu,IOag}(residualcap_{IOag} + residuallab_{IOag}) \\ - \sum_{i} \left[INDMAP_{i,IOag}(indfencingcosts_{fmu,i} + indplancosts_{fmu,i}) \right] \end{aligned} \tag{B.406}$$

$$fmushareagindva_{fmu,IOag} = \frac{fmuagindva_{fmu,IOag}}{\sum_{fmu} fmuagindva_{fmu,IOag}} \tag{B.407}$$

$$fmushareindvaagri_{fmu,i} = \sum_{IOag} (fmushareagindva_{fmu,IOag}\, INDMAP_{i,IOag}) \tag{B.408}$$

$$\begin{aligned} dfmuindva_{fmu,i} = (FMUSHAREEMP_{fmu,i}\, OVERRIDEMAP_{i} + fmushareindvaagri_{fmu,i}) \\ \times dindvalueadded_{dr=DReg1,i} \end{aligned} \tag{B.409}$$

$$fmuindustryva2017_{fmu,i} = dfmuindva_{fmu,i}\, GDPSCALAR \tag{B.410}$$

$$fmureportindva_{fmu,ri} = \sum_{i}(fmuindustryva2017_{fmu,i}\, REPORTINDMAP_{i,ri}) \tag{B.411}$$

$$scaledhhldincome_{dr} = actualhhldincome_{dr}\, GDPSCALAR \tag{B.412}$$

$$fmushareindempagri_{fmu,IOag} = \frac{fmufarmlabIOind_{fmu,IOag}}{\sum_{fmu} fmufarmlabIOind_{fmu,IOag}} \tag{B.413}$$

$$fmushemploy_{fmu,i} = \sum_{IOag}(fmushareindempagri_{fmu,IOag}\, INDMAP_{i,IOag}) + FMUSHAREEMP_{fmu,i} \times OVERRIDEMAP_{i} \tag{B.414}$$

$$fmuindemployment_{fmu,i} = \frac{factorsu_{h=LAB,dr=DReg1,i}}{LSFCONVERTIND_{dr=DReg1,i}}\, fmushemploy_{fmu,i} \tag{B.415}$$

$$fmureportindemp_{fmu,ri} = \sum_{i}(fmuindemployment_{fmu,i}\, REPORTINDMAP_{i,ri}) \tag{B.416}$$

$$zoneagindcap_{ez,agi} = \sum_{ft}(zonefarmcapital_{ez,ft}\, MAPTOAGIND_{ft,agi}) \tag{B.417}$$

$$zoneagindlab_{ez,agi} = \sum_{ft}(zonefarmlab_{ez,ft}\, MAPTOAGIND_{ft,agi}) \tag{B.418}$$

$$taagindcap_{ta,agi} = \sum_{ez}(zoneagindcap_{ez,agi}\, TAZONEMAP_{ta,ez,agi}) \tag{B.419}$$

$$taagindlab_{ta,agi} = \sum_{ez}(zoneagindlab_{ez,agi}\, TAZONEMAP_{ta,ez,agi}) \tag{B.420}$$

$$\begin{aligned} taioagindcap_{ta,IOag=IOAg01} &= taagindcap_{ta,agi=AgIn01} + taagindcap_{ta,agi=AgIn02} + taagindcap_{ta,agi=AgIn05} \\ &\quad + taagindcap_{ta,agi=AgIn06} \\ taioagindcap_{ta,IOag=IOAg02} &= taagindcap_{ta,agi=AgIn03} \\ taioagindcap_{ta,IOag=IOAg03} &= taagindcap_{ta,agi=AgIn04} \end{aligned} \tag{B.421}$$

$$\begin{aligned} taioagindlab_{ta,IOag=IOAg01} &= taagindlab_{ta,agi=AgIn01} + taagindlab_{ta,agi=AgIn02} + taagindlab_{ta,agi=AgIn05} \\ &\quad + taagindlab_{ta,agi=AgIn06} \\ taioagindlab_{ta,IOag=IOAg02} &= taagindlab_{ta,agi=AgIn03} \\ taioagindlab_{ta,IOag=IOAg03} &= taagindlab_{ta,agi=AgIn04} \end{aligned} \tag{B.422}$$

$$ioindfencingplancosts_{fmu,IOag} = \sum_{i}\left[INDMAP_{i,IOag}(indfencingcosts_{fmu,i} + indplancosts_{fmu,i})\right] \tag{B.423}$$

$$taioagindva_{ta,IOag} = taioagindcap_{ta,IOag} + taioagindlab_{ta,IOag} + (residualcap_{IOag} + residuallab_{IOag}) \times \sum_{fmu}(FMUSHRESIDUAL_{fmu,IOag}\, FMUTAMAP_{fmu,ta,IOag}) - \sum_{fmu}(ioindfencingplancosts_{fmu,IOag}\, FMUTAMAP_{fmu,ta,IOag}) \quad (B.424)$$

$$tashareindvaagri_{ta,i} = \sum_{IOag}(tashareioagindva_{ta,IOag}\, INDMAP_{i,IOag}) \quad (B.425)$$

$$dtaindva_{ta,i} = dindvalueadded_{dr=DReg1,i}\ (TASHAREEMP_{ta,i}\, OVERRIDEMAP_i + tashareindvaagri_{ta,i}) \quad (B.426)$$

$$taindustryva2017_{ta,i} = dtaindva_{ta,i}\, GDPSCALAR \quad (B.427)$$

$$tareportindval_{ta,ri} = \sum_i(taindustryva2017_{ta,i}\, REPORTINDMAP_{i,ri}) \quad (B.428)$$

$$tashareioagriemp_{ta,IOag} = \frac{taioagindlab_{ta,IOag}}{\sum_{ta} taioagindlab_{ta,IOag}} \quad (B.429)$$

$$tashemploy_{ta,i} = \sum_{IOag}(tashareioagriemp_{ta,IOag}\, INDMAP_{i,IOag}) + TASHAREEMP_{ta,i}\, OVERRIDEMAP_i \quad (B.430)$$

$$taindemployment_{ta,i} = \frac{factorsu_{h=LAB,dr=DReg1,i}}{LSFCONVERTIND_{dr=DReg1,i}}\, tashemploy_{ta,i} \quad (B.431)$$

$$tareportindemp_{ta,ri} = \sum_i(taindemployment_{ta,i}\, REPORTINDMAP_{i,ri}) \quad (B.432)$$

B.15 Scenario setting equations

B.15.1 Auxiliaries

$$addtravelcosts_{dr} = \sum_c \left(ADDHHLDTRAVEL_{dr,c}(t)\, \mathbf{Pcompcommd}_{dr,c}\right) \quad (B.433)$$

$$maxprod_{dr,i} = \left(qasplannedprod_{dr,i} \times OPERABILITY_{sr\to dr,i}(t)\right) Pcindustrys_{sr\to dr,i} \quad (B.434)$$

$$qasplannedprod_{dr,i} = \max\left(qdesiredprod_{dr,i}(t = SHOCKINITIATION), qdesiredprod_{dr,i}\right) \quad (B.435)$$

$$qdesiredprod_{dr,i} = \frac{\mathbf{Desiredprod}_{dr,i}}{Pcindustrys_{sr\to dr,i}} \quad (B.436)$$

$$dommarginq_{dr} = \sum_{sr}\sum_c (regcdomcomms_{sr,dr,c}\, DMARGINSHOCKCOEF_{sr,dr,c}(t)) \quad (B.437)$$

$$exportmargindemand_{sr,m} = \sum_{c} \left(expcommoditys_{sr,c}\, EMARGINSHOCKCOEF_{sr,c,m}(t)\right) \tag{B.438}$$

$$exportmarginsupply_{dr,m} = \sum_{sr} \left(exportmargindemand_{sr,m}\, REGSHEXPMAR_{sr,dr,m}(t)\right) \tag{B.439}$$

$$importmargindemand_{dr,m} = \sum_{c} \left(\mathbf{Estimports}_{dr,c}\, IMARGINSHOCKCOEF_{dr,c,m}(t)\right) \tag{B.440}$$

$$importmarginsupply_{sr,m} = \sum_{dr} \left(importmargindemand_{dr,m}\, REGSHIMPMAR_{sr,dr,m}(t)\right) \tag{B.441}$$

$$\begin{aligned} marginconsumpq_{dr,c} = & \left(exportmarginsupply_{dr,m=Road} + importmarginsupply_{sr\to dr,m=Road}\right. \\ & \left. + dommarginq_{dr}\right) \times ROADMAP_c \\ & + \left(exportmarginsupply_{dr,m=Rail} + importmarginsupply_{sr\to dr,m=Rail}\right) \\ & \times RAILMAP_c \end{aligned} \tag{B.442}$$

$$pimprailmargins_{sr,dr} = \sum_{c} \left(\mathbf{Pregdomcomm}_{sr,dr,c}\, RAILMAP_c\right) \tag{B.443}$$

$$pimproadmargins_{sr,dr} = \sum_{c} \left(\mathbf{Pregdomcomm}_{sr,dr,c}\, ROADMAP_c\right) \tag{B.444}$$

$$\begin{aligned} pimportmargins_{dr,c} = & \left(\sum_{sr} \left(REGSHIMPMAR_{sr,dr,m=Road}(t)\, pimproadmargins_{sr,dr}\right)\right) \\ & \times IMARGINSHOCKCOEF_{dr,c,m=Road}(t) \\ & + \left(\sum_{sr} \left(REGSHIMPMAR_{sr,dr,m=Rail}(t)\, pimprailmargins_{sr,dr}\right)\right) \\ & \times IMARGINSHOCKCOEF_{dr,c,m=Rail}(t) \end{aligned} \tag{B.445}$$

$$pexprailmargins_{dr} = \sum_{c} \left(\mathbf{Pcompcommd}_{dr,c}\, RAILMAP_c\right) \tag{B.446}$$

$$pexproadmargins_{dr} = \sum_{c} \left(\mathbf{Pcompcommd}_{dr,c}\, ROADMAP_c\right) \tag{B.447}$$

$$\begin{aligned} pexportmargins_{sr,c} = & \left(\sum_{dr} \left(REGSHEXPMAR_{sr,dr,m=Road}(t)\, pexproadmargins_{dr}\right)\right) \\ & \times EMARGINSHOCKCOEF_{sr,c,m=Road}(t) \\ & \left(\sum_{dr} \left(REGSHEXPMAR_{sr,dr,m=Rail}(t)\, pexprailmargins_{dr}\right)\right) \\ & \times EMARGINSHOCKCOEF_{sr,c,m=Rail}(t) \end{aligned} \tag{B.448}$$

$$\begin{aligned} pregdomcomminclmargin_{sr,dr,c} = & DMARGINSHOCKCOEF_{sr,dr,c}(t)\, pimproadmargins_{sr,dr} \\ & + \mathbf{Pregdomcomm}_{sr,dr,c} \end{aligned} \tag{B.449}$$

C Adoption of mitigations

C.1 Mitigation adoption in the presence of a regulatory deadline

We wish to model the situation where farmsystem A applies some sort of mitigation option and becomes farmsystem B. These farmsystems will have different income and expenditure items/balances in the economic model. We assume that all farms must be either A or B, so the rate of change will be described as:

$$\frac{dA}{dt} = -[r(t) + d(t)]A$$
$$\frac{dB}{dt} = [r(t) + d(t)]A \tag{C.1}$$

where A is the hectares in farmsystem A, B is the hectares in farmsystem B, $r(t)$ is the rate of adoption in the absence of a regulatory deadline, and $d(t)$ is an additional term to model the impact of the deadline.

To simplify things, we initially consider the case in the absence of any sort of pressure due to regulatory deadlines. Then we consider the deadline term separately.

Adoption curve First, we assume that farmsystem A fully (100%) transitions to farmsystem B following the sigmoidal (S-shaped) technological adoption curve:

$$B = \frac{N}{1 + e^{-k(t-m)}} \tag{C.2}$$

This is the standard logistic function, where k controls the steepness of the S-shaped curve, m gives the midpoint of the curve, and N is the total number of hectares. In this case, all farms must be in A or B, thus $A = N - B$ and:

$$A = N - \frac{N}{1 + e^{-k(t-m)}}$$
$$A = N\left[1 - \frac{1}{1 + e^{-k(t-m)}}\right]$$
$$A = N\left[\frac{1 + e^{-k(t-m)}}{1 + e^{-k(t-m)}} - \frac{1}{1 + e^{-k(t-m)}}\right]$$
$$A = \frac{N\, e^{-k(t-m)}}{1 + e^{-k(t-m)}} \tag{C.3}$$

Next, what we need is the *rate* at which A decreases, or conversely that B grows at. We can differentiate Eq. C.2 to give:

$$\frac{dB}{dt} = \frac{kN\, e^{-k(t-m)}}{[1 + e^{-k(t-m)}]^2} \tag{C.4}$$
$$\tag{C.5}$$

And we know that $\frac{dB}{dt} = -\frac{dA}{dt}$, and so:

$$\frac{dA}{dt} = \frac{-kN\, e^{-k(t-m)}}{[1 + e^{-k(t-m)}]^2} \tag{C.6}$$

Whilst we could implement these two equations directly in our dynamic model, it would only work for the case where all $A \to B$ and there are no other changes to A or B. Intuitively, we want the rate to be given in terms of the hectares still in farmsystem A, so that when this goes to zero, there is no further change, and if this increases, the rate will increase. In other words, we want something in this form:

$$\frac{dA}{dt} = -r(t)A \tag{C.7}$$

where $r(t)$ is an adoption rate, or *rate of change* of A.

Comparing Eqs. C.6 and C.3, we see that we can write:

$$\frac{dA}{dt} = \frac{-k}{1+e^{-k(t-m)}}A \tag{C.8}$$

Which matches the form we want, Eq. C.7, if we set:

$$r(t) = \frac{k}{1+e^{-k(t-m)}} \tag{C.9}$$

Here the rate is the standard logistic (sigmoidal) function, multiplied by k (so this function will go from 0 to k, instead of 0 to 1).

If we make the assumption that there will be no transitions before the announcement time, we get the extra condition:

$$r(t) = \begin{cases} 0 & \text{for} \quad t \leq t_a \\ \frac{k}{1+e^{-k(t-m)}} & \text{for} \quad t_a < t \end{cases} \tag{C.10}$$

The rate function for different values of k and m are shown in Figure C.1. If we implement this in a dynamic model we get S-shaped curves for A and B as expected (shown in Figure C.3).

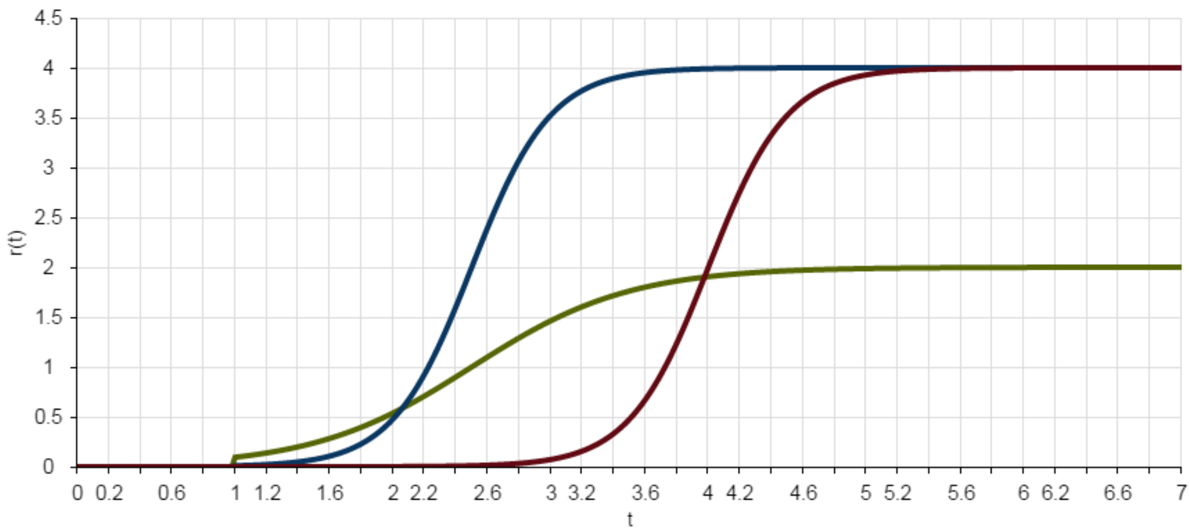

Figure C.1 Adoption rates $r(t)$ from Eq. C.10, for three different cases with a policy announcement time of $t_a = 1$: k=2, m=2.5 (green), k=4, m=2.5 (blue), and k=4, m=4 (burgundy).

Regulatory deadline curve In the above, we have assumed that there is total transition, but depending on the k and m values chosen, it may take a long while. If we want to simulate the enforcement of a regulation, we need to add a regulatory deadline term, $d(t)$, see Eq. C.1. There are a few options for this, but the simplest one is:

$$d(t) = \frac{1}{t_d - t} \qquad \text{for} \quad t < t_d \tag{C.11}$$

where t_d is the time of the deadline. It makes sense (we think) to have this term being zero before the policy is announced (t_a), and then to increase during the notice period ($t_d - t_a$). To achieve this we can set:

$$d(t) = \begin{cases} 0 & \text{for} \quad t \leq t_a \\ \frac{1}{t_d - t} - \frac{1}{t_d - t_a} & \text{for} \quad t_a < t < t_d \\ L & \text{for} \quad t_d \leq t \end{cases} \tag{C.12}$$

where L is a large number that forces $A \to B$ very quickly once the deadline is reached.

With a small time step this can mean that just before the deadline the deadline pressure goes to infinity, which is not ideal numerically. We also might want to stretch the impact of the deadline pressure so that it is felt sooner, or so that it is only close to the deadline. We can think of that as the 'urgency' people feel relating to the deadline. These two considerations give us:

$$d(t) = \begin{cases} 0 & \text{for} \quad t \leq t_a \\ \min\left(c\left(\frac{1}{t_d - t} - \frac{1}{t_d - t_a}\right), L\right) & \text{for} \quad t_a < t < t_d \\ L & \text{for} \quad t_d \leq t \end{cases} \tag{C.13}$$

Where $c = 1$ gives us the deadline pressure we had before, $c > 1$ means there is pressure to change sooner, and $0 < c < 1$ means that the pressure kicks in later.

This deadline curve, Eq. C.13, for some chosen parameter values is shown in Figure C.2.

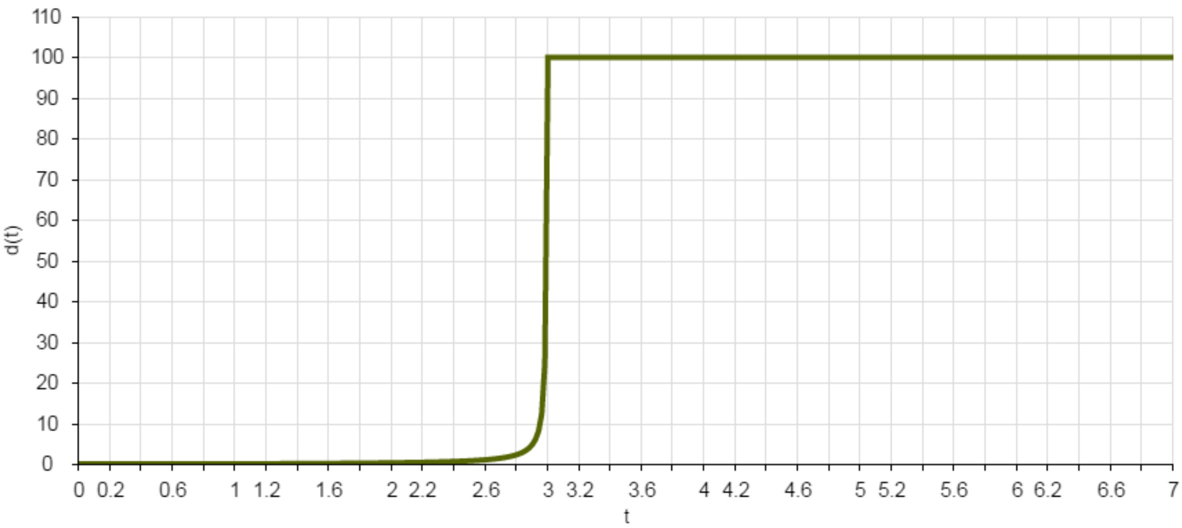

Figure C.2 Deadline pressure $d(t)$ from Eq. C.13, with the deadline at $t_d = 3$, the maximum rate at $L = 100$ and the 'urgency' modifier $c = 0.5$.

Example results Results when the adoption curves are used but there is no regulatory deadline enforced are shown in Figure C.3.

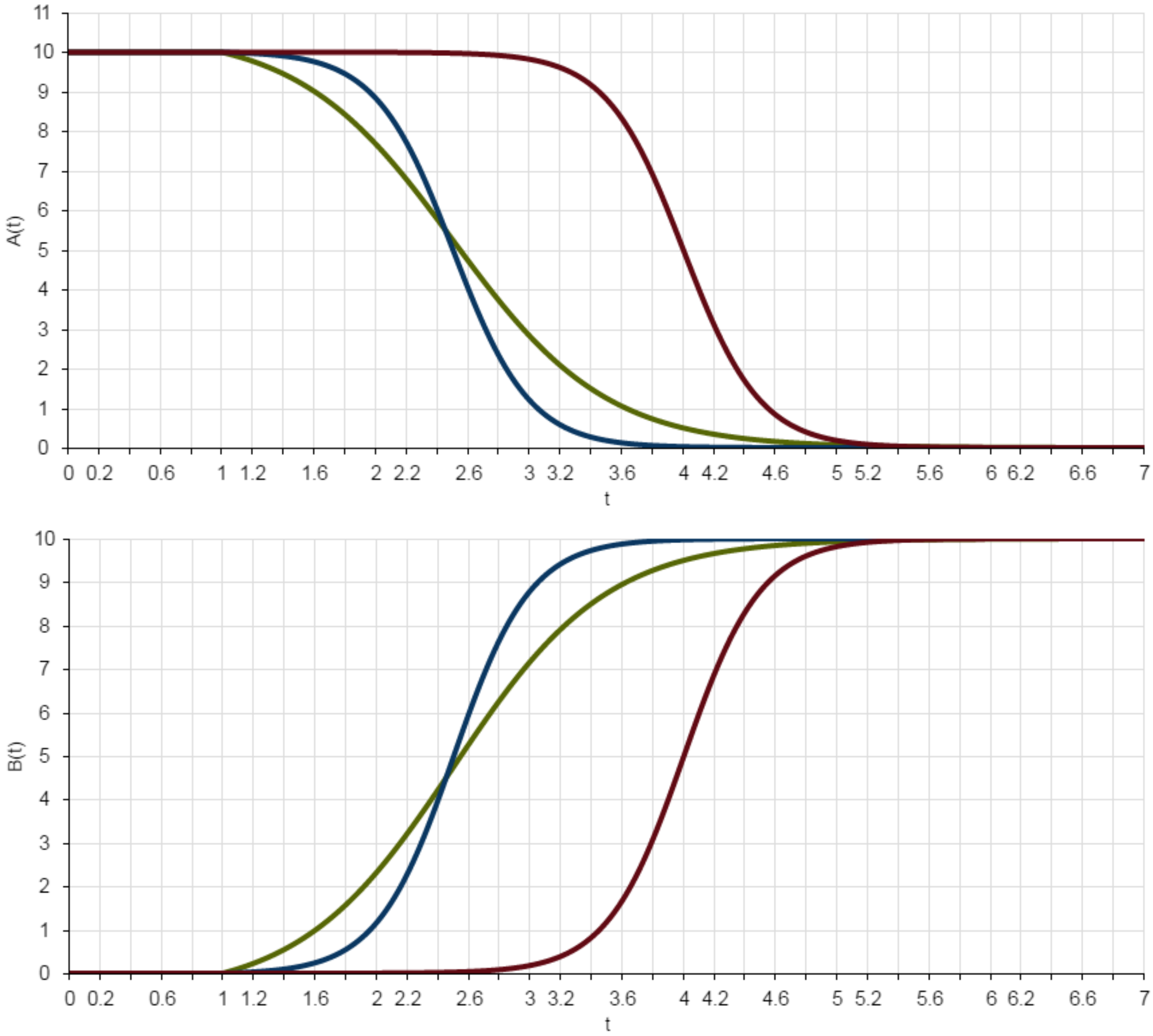

Figure C.3 Number of hectares of farm system A and B for three different cases from Figure C.1, with $N = A(0) = 10$.

Results when the adoption curves and the deadline pressure are included are shown in Figure C.4.

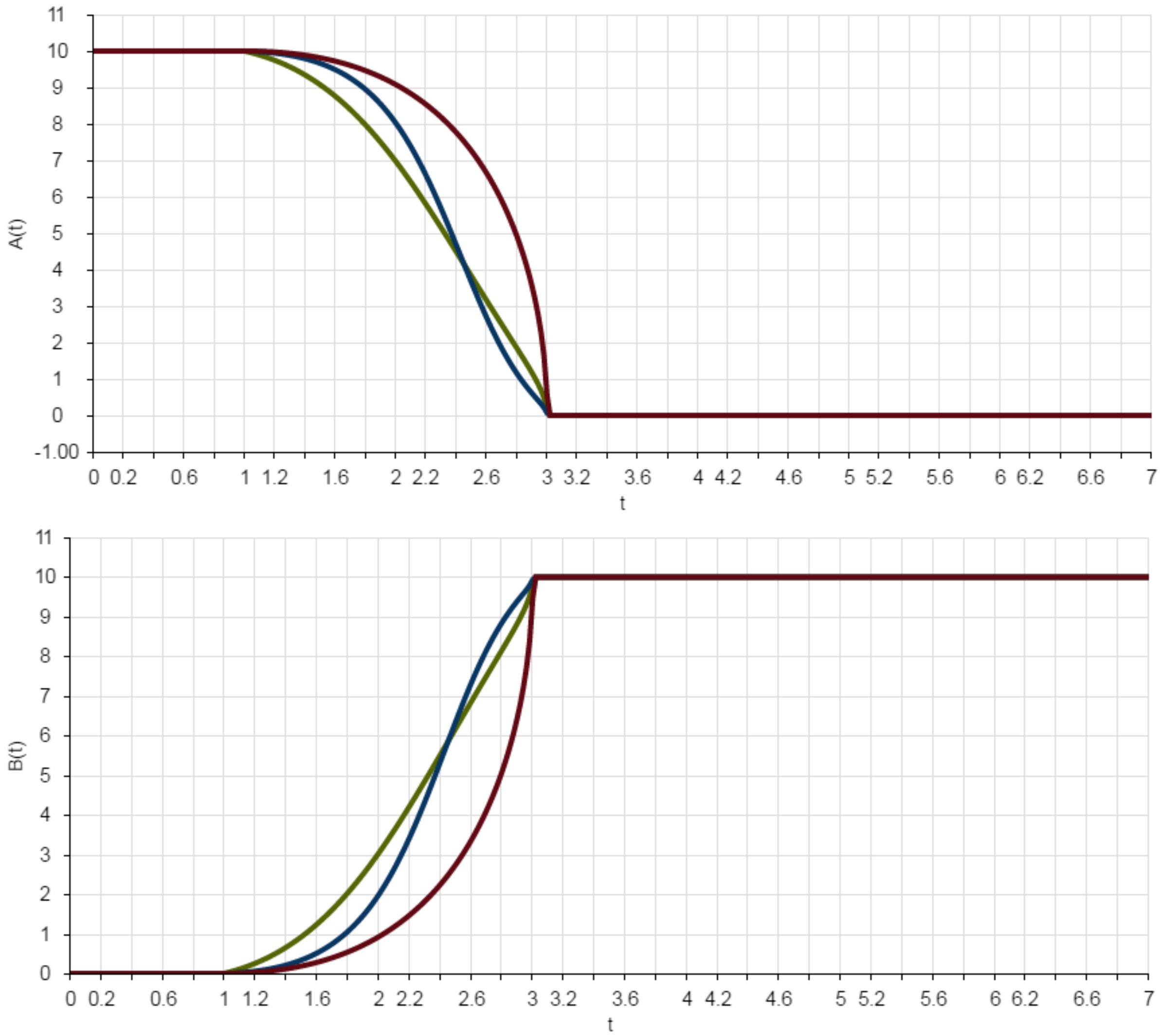

Figure C.4 Number of hectares of farm system A and B for three different cases from Figure C.1 and with the deadline pressure applied and $N = A(0) = 10$.

C.2 Notes/considerations

- We have assumed that there is total transition from A to B.
- If the incentives/reasons to change from A to B are low, transition wouldn't happen before the deadline. All change will be driven by the deadline term.
- We haven't explicitly assumed that $A = N$ and $B = 0$ at t_a, so this should work in a range of situations. The main driving factor is how many hectares are in farmsystem A.
- If there are different possibilities (e.g. farmsystem A could change to farmsystem B or farmsystem C) then we will need to have a bit of a think about how to adjust these equations, as here we have assumed that $A + B = N$. It should still work, but we haven't covered that scenario here, so care must be taken.
- Here we set the deadline curve to a maximum rate L. But as this is added to the adoption rate, $r(t)$, then in practice it might be better to set:

  $$\begin{aligned} \frac{dA}{dt} &= -\left[\max(r(t) + d(t), L)\right] A \\ \frac{dB}{dt} &= \left[\max(r(t) + d(t), L)\right] A \end{aligned} \tag{C.14}$$

  In which case, we could use the form of Eq. C.12, instead of Eq. C.13 for $d(t)$.
- If the maximum rate L is set too high, then there is nothing stopping the stock A becoming negative. If we are using Euler's method to solve then we just chose the maximum rate as $L \leq 1/\delta t$ but with RK this does not work. In InsightMaker and Stella, it is possible to just select that no Stocks (land areas) can go negative, but this is not possible in Vensim.